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THE HYSTERESIS OF IRON AND STEEL AT ORDINARY TEMPERATURES AND AT THE TEMPERATURE OF SOLID CARBON DIOXIDE.

By A. H. Thiessen.

THE present article has to do with some determinations of the hysteresis of iron, steel, and nickel steel. A comparative study was made of the hysteresis of these metals at three different temperatures, one of these being the low temperature reached by means of solid CO_2, and at different values of the maximum magnetizing force. Owing to the peculiar magnetic properties of nickel steel a specimen of this metal was tested, but at only one value of the maximum induction.

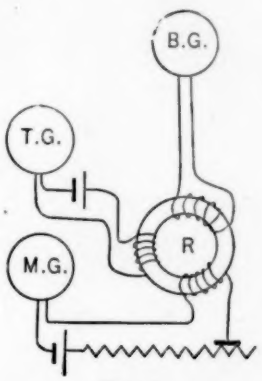

Fig. 1.

The ballistic method was used both in determining curves of magnetization and in finding the losses due to hysteresis. A diagram of the apparatus used is shown in Fig. 1. *R* is the ring of the metal to be tested. *BG* is a ballistic galvanometer employed in determining the magnetic induction, and is connected with the secondary coil of the ring. *MG* is a Moler galvanometer to measure current, and is connected in circuit with the primary coil, a source of current, and a variable resistance.

To determine the temperature of the metal a coil of copper wire

was wound next to the surface of the ring. The resistance of this coil was compared with the resistance of a standard coil (kept at constant temperature) by the method of fall of potential. *TG* in the diagram represents the galvanometer used for this purpose.

Curves of hysteresis were taken with the metal at three different temperatures. Readings were first taken at the temperature of the room, or about 20° C.; then at the boiling point of water; and lastly at such low temperatures as could be obtained with solid carbon dioxide.

When the readings were taken at ordinary temperatures the ring was immersed in a bath of oil, and the temperature was thus kept constant. At the boiling point the metal ring was immersed in a vessel containing glymol, and this in turn was placed in boiling water.

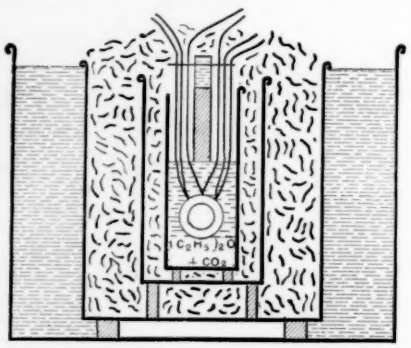

Fig. 2.

At the low temperatures the apparatus shown in Fig. 2 was used to cool the rings. As is seen in the figure it consisted of four vessels. The inner contained solid carbon dioxide and ether. A long wooden stopper was fitted tightly in this, and the terminals of the coils were led through this stopper. By means of these terminal wires the ring was suspended in the vessel. A small hole was bored in the wooden stopper to admit the ether and carbon dioxide; this was closed by a piece of cork. Around this vessel were two jackets of mineral wool, which was torn in shreds and laid lightly in. Outside of these was a jacket of ice and water. Mineral wool was then laid on top of all to insure thermal insulation.

The metal to be tested was turned in the shape of a ring with a cross section of about one square centimeter, and was about five centimeters in diameter. Four different rings were worked with, which are referred to as *A*, *B*, *C*, and *D*. *A* and *B* were from the same piece of soft wrought iron, the difference between them being that the cross section of *B* was nearly circular, while that of ring *A*

was square. The results obtained were practically the same, but the data are given only for ring *A*. Ring *C* was made of crescent tool steel. Ring *D* was of 5% nickel steel. All specimens were annealed before being tested. The cross sections of *C* and *D* were rectangular, but oblong, with the longer side perpendicular to the plane of the ring. The number of turns per centimeter on the outside was thus more nearly the same as on the inside, and hence the magnetization was more uniform. This shape of the ring was also better in regard to heating or cooling.

The specimens, in all cases, were turned so that the grain of the metal ran at right angles to the plane of the ring; in the case of the figure (see Fig. 3) at right angles to the plane of the paper. The magnetization, then, if it was affected at all by the grain of the metal, was uniform throughout the ring.

Fig. 3.

Before winding, the rings were shellacked; a layer of silk was then wrapped about the ring, and the whole was again shellacked to insure complete insulation. A layer of No. 29 copper wire was next wound upon the ring, this coil being used to measure the temperature, as described above. The secondary and primary coils were then wound, with the secondary lying between the layers of the primary. After each layer had been wound the rings were shellacked and baked.

In order to be able to immerse the rings directly in the ether and carbon dioxide they were coated with gutta that had been dissolved in carbon bisulphide, the rings being immersed in this solution and then allowed to dry; the carbon bisulphide passed off, leaving a coating of gutta upon the rings. Since ether does not appreciably act upon the gutta, the shellac was protected and all danger of short circuit was avoided. At very low temperatures, however, the gutta is brittle, and additional caution must be used to preserve the coating intact.

In taking the data for the hysteresis loops, the same maximum magnetizing force was used for all three temperatures. In the case of the wrought iron three sets, and of tool steel two sets of hysteresis loops were taken. The first set corresponds to such a maximum magnetizing force as would produce magnetization near saturation,

while the second and third sets were taken with a magnetizing current of lower strength.

As already mentioned, three different specimens of metals were experimented with, iron, steel, and nickel steel. Numerical data are given in the tables at the end of this article. A graphical

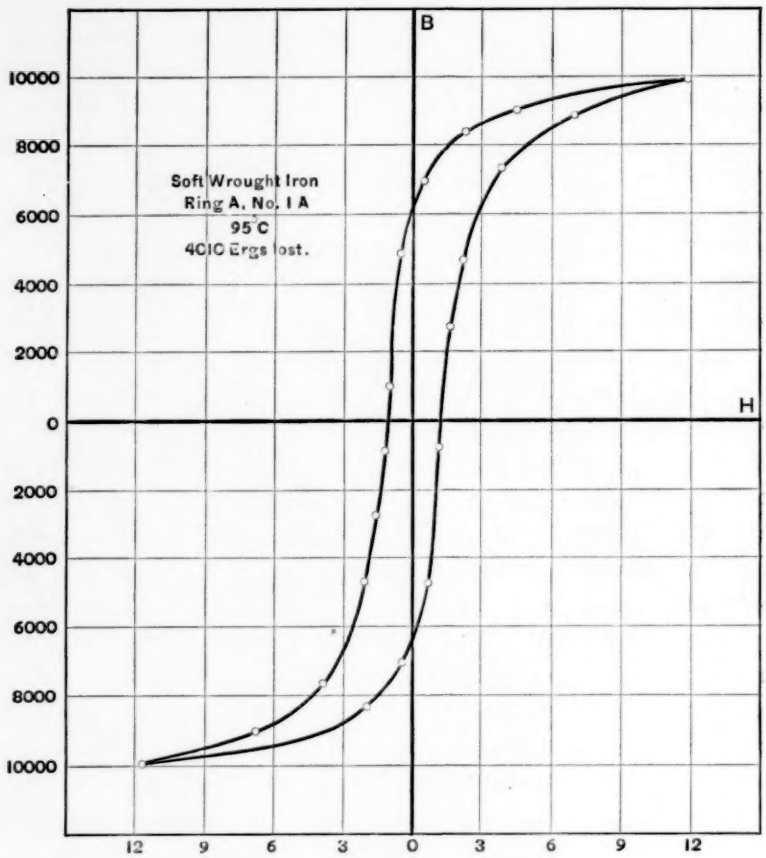

Fig. 4.

representation of the results obtained with wrought iron is shown in Figs. 4 to 7 inclusive. By comparing Fig. 4 with Fig. 5 it may be seen that the losses increase with decreasing temperatures when the maximum magnetizing current produces approximate saturation. But this is not true with low magnetizing forces, as is shown in Figs. 6 and 7. In these two cases, with such

low values of H as were used, the losses *decreased* with decreasing temperatures; *i. e.*, the behavior was just the opposite of that in the first case.

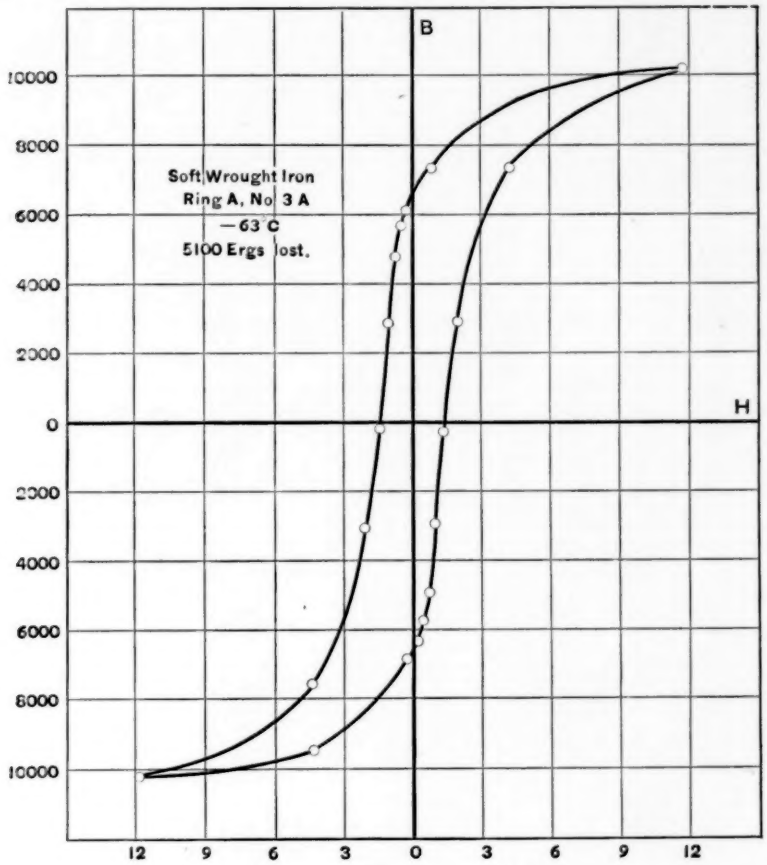

Fig. 5.

The second specimen experimented with was crescent tool steel, annealed. The results are exhibited in Figs. 8 and 9. In Fig. 8 the data were taken under circumstances analogous with those corresponding to Figs. 4 and 5; that is with induction near saturation. In this case, also, it is seen that the hysteresis losses increase with decreasing temperatures. With this specimen, as with the preceding one, the reverse rule is true when H is small, shown in Fig. 9.

The graphical representation of the data taken with nickel steel

is shown in Fig. 10. With this specimen the same conclusions may be drawn as with the other two.

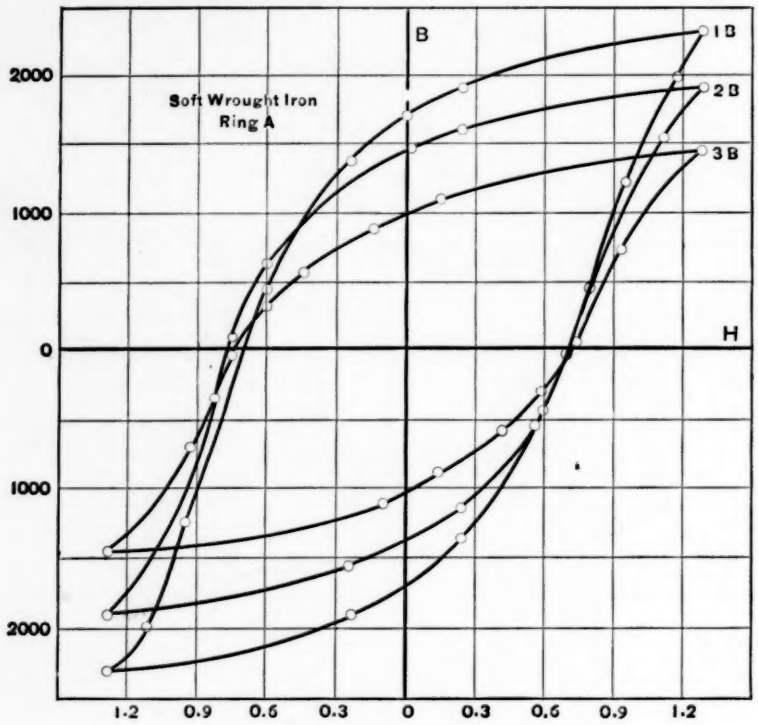

Fig. 6.

No. 1 *B*, 97°C., 433 ergs lost.
No. 2 *B*, 21°C., 371 ergs lost.
No. 3 *B*, −78°C., 266 ergs lost.

The areas of these loops were obtained with a planimeter, and the quantitative results are shown in the following tabulation:

TABLE I.

Soft Wrought Iron.					
$H=11.9$		$H=2\,60.$		$H=1.30$	
Temp.	Losses, Ergs.	Temp.	Losses, Ergs.	Temp.	Losses, Ergs.
95°	4010	97°	1710	97°	430
20°	4580	22°	1610	21°	370
−63°	5100	−80°	1530	−78°	270

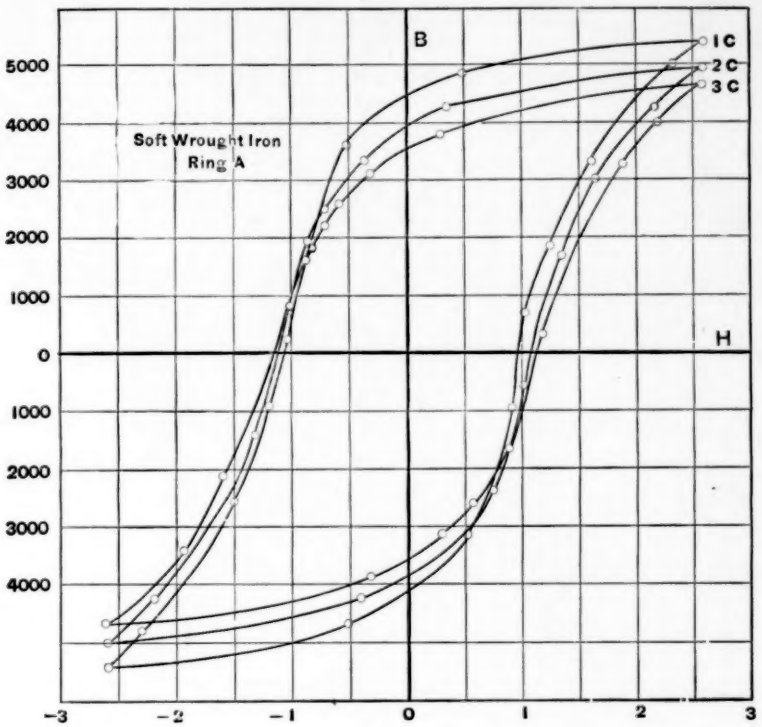

Fig. 7.

No. 1 *C*, 97°C., 1710 ergs lost.
No. 2 *C*, 22°C., 1610 ergs lost.
No. 3 *C*, −80°C., 1530 ergs lost.

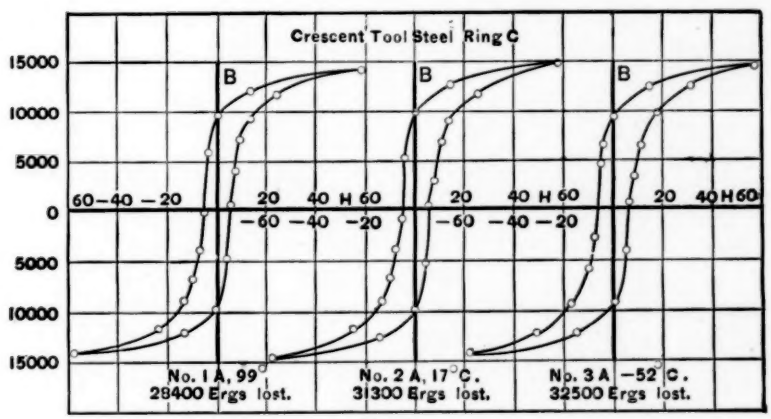

Fig. 8.

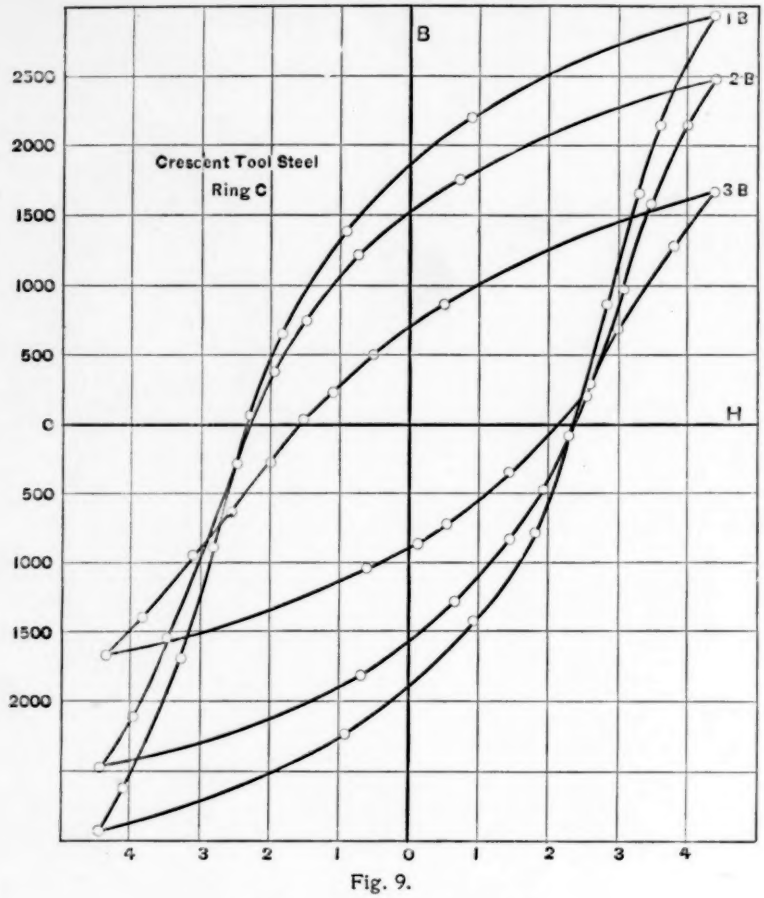

Fig. 9.

No. 1 *B*, 98°C., 1650 ergs lost.
No. 2 *B*, 18°C., 1370 ergs lost.
No. 3 *B*, −55°C., 690 ergs lost.

Crescent Tool Steel.				5 per cent. Nickel Steel.	
$H=57.6$		$H=4.40$		$H=52.9$	
Temp.	Losses.	Temp.	Losses.	Temp.	Losses.
99°	28400	99°	1650	99°	36600
17°	31400	18°	1380	24°	41900
−52°	32600	−55°	700	−65°	44400

The conclusions to be derived from the foregoing graphs and from the tabulation are shown graphically in Figs. 11 to 15 inclu-

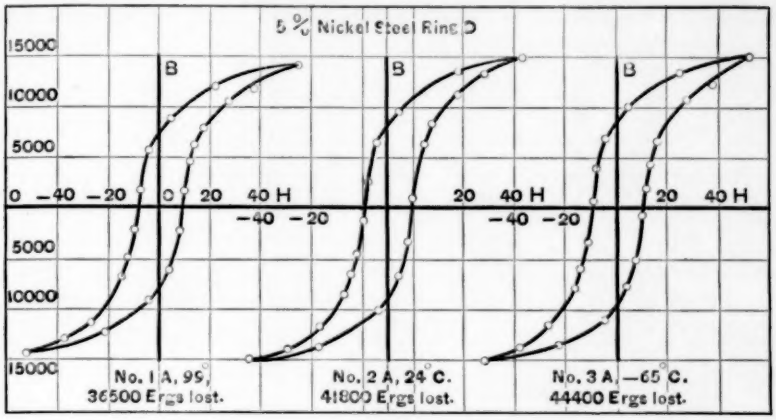

Fig. 10.

sive. They show in general that hysteresis losses increase with decreasing temperatures when the maximum magnetizing force causes approximate magnetic saturation, and that the reverse is true for low magnetizing forces.

The results of such an investigation as this are of more value if the losses at different temperatures are compared when the maximum *induction* in each case is the same. In the experiments, however, it was much more convenient to keep the maximum magnetizing *force* the same. In order to compare the losses for a given value of the maximum induction, Steinmetz's law was resorted to, viz.:

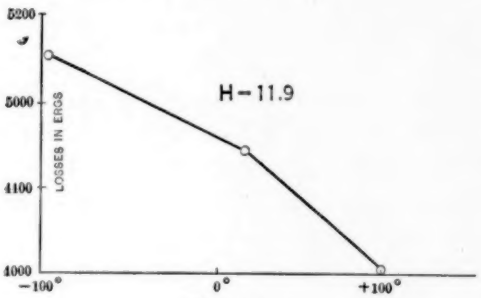

Fig. 11.
Soft wrought iron.

$$W = nB^k.$$

Where W is the hysteresis loss corresponding to a cycle whose maximum induction is B; n and k are constants for the material in question, so long as the temperature remains the same. This law was found to hold in the case of observations taken at the same temperature, and it was therefore possible to find the value of W

for a given maximum B by interpolation. The results in the case of wrought iron are given in Table II.

TABLE II.

Soft Wrought Iron. Ring A.			
Maximum Induction.	Hysteresis Loss per Cycle.		
B	$T = -70°$ C	$T = 20°$ C	$T = 100°$ C
2000	423 Ergs.	397 Ergs.	333 Ergs.
5000	1720 "	1620 "	1520 "
10000	5070 "	4600 "	4030 "

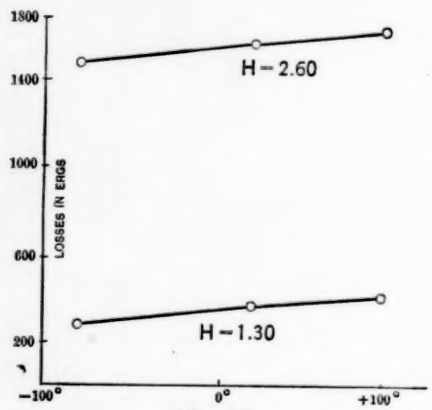

Fig. 12
Soft wrought iron.

In the case of the crescent tool steel (Ring C) the hysteresis losses in ergs per cycle, *for a maximum induction of 14,700*, were found to be: 33,850, at −52°C.; 31,380, at 17°C., and 29,600, at 99°C.

With the 5% Nickel steel (Ring D) the losses per cycle, *for a maximum induction of 14,900*, were: 43,070, at −65°C.; 41,860, at 23°.5C.; 39,700, at 99°.5C.

In each of the last two cases the computations were made upon the assumption that $k = 1.6$. For the wrought iron the data were sufficient to determine the value of k at each of the three temperatures.

It will be observed that when hysteresis loops are taken in which the *maximum induction* is the same for different temperatures, the hysteresis loss is in all cases less at the higher temperatures. This result is in agreement with previous determinations of hysteresis losses at high temperatures.

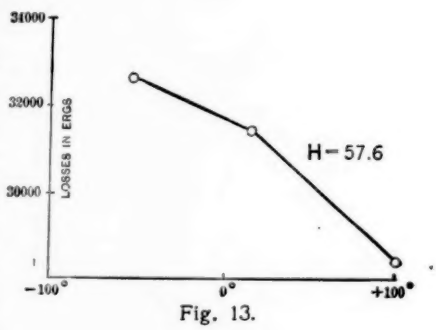

Fig. 13.
Tool steel.

It is interesting to note that the above conclusion is not in agreement with the results of Fleming and Dewar,[1] who have made an extensive series of measurements of the hysteresis losses in soft Swedish iron at ordinary temperatures and at the temperature of liquid air (— 185°C.). The method employed by these experimenters required the use of an alternating current with a frequency of 100 periods per second. No difference could be detected in the hysteresis loss per cycle, for a given maximum induction, at the two temperatures. It is to be observed that the method employed by Fleming and Dewar is susceptible of error on account of the possible presence of Foucault currents, and that its results would also be affected by a "time lag" if such were present. So far as I know the hysteresis of iron has not been investigated for the range of temperatures lying between — 80°C. and — 185°C. If the curve connecting hysteresis loss with temperature formed a maximum somewhere between — 80°C. and — 185°C. the discrepancy between the results of Fleming and Dewar and those given in the present paper would be explained.

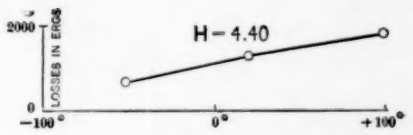

Fig. 14.

Tool steel.

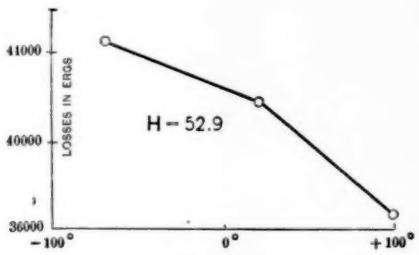

Fig. 15.

5 % Nickel steel.

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[1] Proc. Roy. Soc., Vol. 60, p. 81, 1896.

TABLE III.

Soft Wrought Iron, 21° C.		Soft Wrought Iron, 95° C.		Soft Wrought Iron, −63° C.	
H	*B*	*H*	*B*	*H*	*B*
11.9	9950	11.9	9930	11.9	10200
5.5	8340	2.2	8410	1.0	7300
.5	7050	.6	7060	.2	6830
− .5	5080	− .6	4890	− .2	6130
− 1.0	1960	− 1.0	990	− .4	5680
− 1.2	− 120	− 1.2	− 930	− .7	4810
− 1.5	−1880	− 1.6	−2660	− 1.0	2680
− 2.1	−4300	− 2.3	−4880	− 1.4	− 40
− 2.9	−5950	− 3.9	−7630	− 2.1	− 3060
− 3.8	−7320	− 6.9	−9080	− 4.4	− 7570
− 6.4	−9010	−11.8	−9960	−11.8	−10200
−11.6	−9950	− 2.2	−8360	− 4.4	− 9400
− 2.2	−8410	− .6	−7010	− .2	− 6840
− .5	−6960	.6	−4720	.2	− 6400
.5	−5060	.9	− 820	.4	− 5710
.9	−1990	1.2	1200	.7	− 4900
1.2	30	1.6	2650	1.0	− 2960
1.5	1750	2.3	4750	1.4	− 70
2.2	4170	3.9	7370	2.1	2980
3.8	7140	6.9	8850	4.4	7420
6.8	8780	11.8	9870	11.9	10100
11.9	9810				

TABLE IV.

Soft Wrought Iron, 21° C.		Soft Wrought Iron, 97° C.		Soft Wrought Iron, −78° C.	
H	*B*	*H*	*B*	*H*	*B*
1.28	1920	1.29	2320	1.28	1460
.24	1610	.23	1930	.14	1110
− .24	1260	0	1710	− .14	880
− .59	640	− .23	1380	− .43	570
− .72	110	− .63	470	− .60	330
− .82	− 340	− .79	− 440	− .76	− 40
−1.10	−1500	− .83	− 650	− .93	− 710
−1.29	−1920	− .95	−1240	−1.29	−1460
− .25	−1570	−1.17	−2000	− .14	−1110
.25	−1150	−1.29	−2320	.14	− 880
.57	− 550	− .23	−1920	.44	− 560
.72	− 10	.23	−1370	.60	− 320
.82	420	.60	− 440	.74	60
1.12	1550	.79	460	.93	730
1.29	1950	.85	670	1.29	1490
		.96	1240		
		1.18	2000		
		1.30	2300		

TABLE V.

Soft Wrought Iron, 22° C.		Soft Wrought Iron, 97° C.		Soft Wrought Iron, −80° C.	
H	*B*	*H*	*B*	*H*	*B*
2.59	4980	2.59	5430	2.62	4670
.35	4250	.50	4850	.31	3830
− .35	3340	− .50	3670	− .31	3130
− .71	2500	− .86	1840	− .57	2610
− .86	1930	−1.03	270	− .71	2210
−1.01	810	−1.20	− 880	− .89	1640
−1.32	−1410	−1.51	−2540	−1.18	− 360
−1.66	−2850	−1.72	−3110	−1.54	−2080
−2.18	−4220	−2.31	−4790	−1.94	−3390
−2.58	−4990	−2.59	−5430	−2.59	−4670
− .41	−4220	− .51	−4680	− .31	−3850
.41	−3240	.51	−3160	.31	−3160
.74	−2350	.91	− 950	.58	−2620
.86	−1750	1.08	720	.88	−1660
1.01	− 560	1.25	1840	1.18	3100
1.35	1700	1.60	3310	1.52	2000
1.65	3060	1.76	3770	1.88	3330
2.17	4260	2.35	5000	2.59	4690
2.57	4880	2.64	5430		

TABLE VI.

Tool Steel, 17° C.		Tool Steel, 99° C.		Tool Steel, −52° C.	
H	*B*	*H*	*B*	*H*	*B*
58.1	14700	57.6	14280	57.9	14350
14.0	12600	12.7	12140	14.8	12320
.0	9800	0	9650	0	9480
− 4.1	5300	− 3.7	5830	− 3.5	6630
− 5.8	− 80	− 5.4	− 210	− 4.5	4665
− 8.0	− 3900	− 7.4	− 3910	− 6.6	0
−10.7	− 6900	−10.0	− 6810	− 8.3	− 2782
−14.2	− 9100	−13.1	− 8910	−11.0	− 5900
−25.0	−11900	−23.6	−11650	−17.6	− 9560
−58.1	−14700	−57.9	−14270	−31.4	−12380
−14.1	−12600	−12.9	−12160	−57.9	−14350
0	− 9800	0	− 9640	−14.7	−12301
4.1	− 5300	3.9	− 4970	0	− 9330
5.8	40	5.4	580	4.6	− 4250
8.0	3000	7.4	4220	6.3	630
10.7	6830	8.7	7060	8.3	3310
14.1	9050	13.0	9160	11.0	6340
24.8	11900	23.6	11930	17.5	9830
58.4	14700	57.9	14550	31.4	12550
				58.3	14450

TABLE VII.

Tool Steel, 18° C.		Tool Steel, 99° C.		Tool Steel, −55° C.	
H	*B*	*H*	*B*	*H*	*B*
4.42	2480	4.42	2940	4.42	1680
.73	1770	.94	2230	.48	850
− .73	1220	− .94	1400	− .48	500
−1.51	760	−1.83	670	−1.10	240
−1.96	390	−2.28	60	−1.48	40
−2.60	− 250	−2.80	− 890	−2.02	− 280
−3.05	− 970	−3.28	−1700	−2.56	− 640
−3.46	−1530	−3.62	−2180	−3.08	− 970
−3.94	−2120	−4.12	−2630	−3.84	−1410
−4.42	−2480	−4.42	−2940	−4.36	−1680
− .66	−1810	− .93	−2240	− .56	−1040
.66	−1280	.93	−1420	.56	− 730
1.47	− 830	1.80	− 690	1.47	− 350
1.94	− 460	2.29	− 100	2.58	290
2.52	200	2.86	880	3.08	680
3.06	970	3.28	1670	3.84	1271
3.46	1560	3.62	2142	4.36	1655
4.03	2150	4.42	2890		
4.42	2510				

TABLE VIII.

Nickel Steel, 23° C.		Nickel Steel, 99° C.		Nickel Steel, −65° C.	
H	*B*	*H*	*B*	*H*	*B*
52.9	14900	52.7	14160	52.9	15180
26.8	13380	21.7	12040	21.9	13250
4.2	9730	4.2	8920	4.2	10170
− 4.2	6550	− 4.2	5750	− 4.2	7010
− 7.8	3100	− 7.6	1800	− 7.7	4240
− 9.9	− 1020	− 9.9	− 2050	− 9.9	703
−12.1	− 4320	−12.0	− 4870	−11.9	− 3020
−14.3	− 6420	−14.1	− 6630	−14.2	− 5540
−17.0	− 8330	−17.0	− 8210	−17.2	− 7770
−26.7	−11640	−26.7	−11130	−26.8	−11630
−38.4	−13500	−37.6	−12830	−38.3	−13710
−53.0	−14900	−52.9	−14160	−53.4	−15180
−26.9	−13390	−21.7	−12090	−21.6	−13230
− 4.2	− 9780	− 4.2	− 8960	− 4.1	−10510
4.2	− 6610	4.2	− 5830	4.1	− 7490
7.7	− 3140	7.7	− 1890	7.5	− 4890
9.7	900	9.9	1910	9.6	− 1415
12.0	4200	11.9	4650	11.7	2110
15.0	6330	13.9	6310	13.8	4490
17.2	8240	16.9	7890	16.6	6610
26.9	11540	26.6	10770	26.4	10383
38.0	13360	37.8	12500	38.3	12410
52.7	14730	53.0	13890	53.6	13820

A CALORIMETRIC DETERMINATION OF ENERGY DISSIPATED IN CONDENSERS.

By Edward B. Rosa and Arthur W. Smith.

In a former paper[1] we gave the results of measurements by means of a wattmeter of the energy dissipated in condensers when they were subjected to an alternating electromotive force. The results were such that we desired to confirm them by a totally independent method; and, in addition, to measure the energy dissipated in some paraffined paper condensers which showed so small a loss that with the coils at our disposal the Resonance Method, employed successfully on beeswax and rosin condensers, would not give sufficiently accurate values. We therefore constructed a special calorimeter for the purpose of measuring the total quantity of heat produced in the condensers, and which represents the total energy dissipated.

Fig. 1 gives an external view of the calorimeter, and Fig. 2 a vertical section. The calorimeter proper, *A*, is the inner of three concentric boxes, and is 33 cm. long, 30 cm. deep, and 10 cm. in breadth. It has a copper lining (*b*), and a copper jacket (*c*), and is protected by the two exterior boxes from fluctuations of temperature without. The general principle of the calorimeter is (1) to prevent any loss or gain of heat through its walls, and (2) to carry away and measure all heat generated within by a stream of water. To effect the first condition, the two concentric copper walls, the lining and the jacket, are maintained, as nearly as possible, at the same temperature. This, of course, will reduce the flow of heat through the intervening wooden wall to a minimum, and make the "cooling correction" small, if not zero.

1. To Eliminate the Cooling and Capacity Correction.

In order to ascertain any difference of temperature between the copper walls *b* and *c*, a differential air thermometer is used. Each

[1] Phys. Rev., ——., ——, 189–.

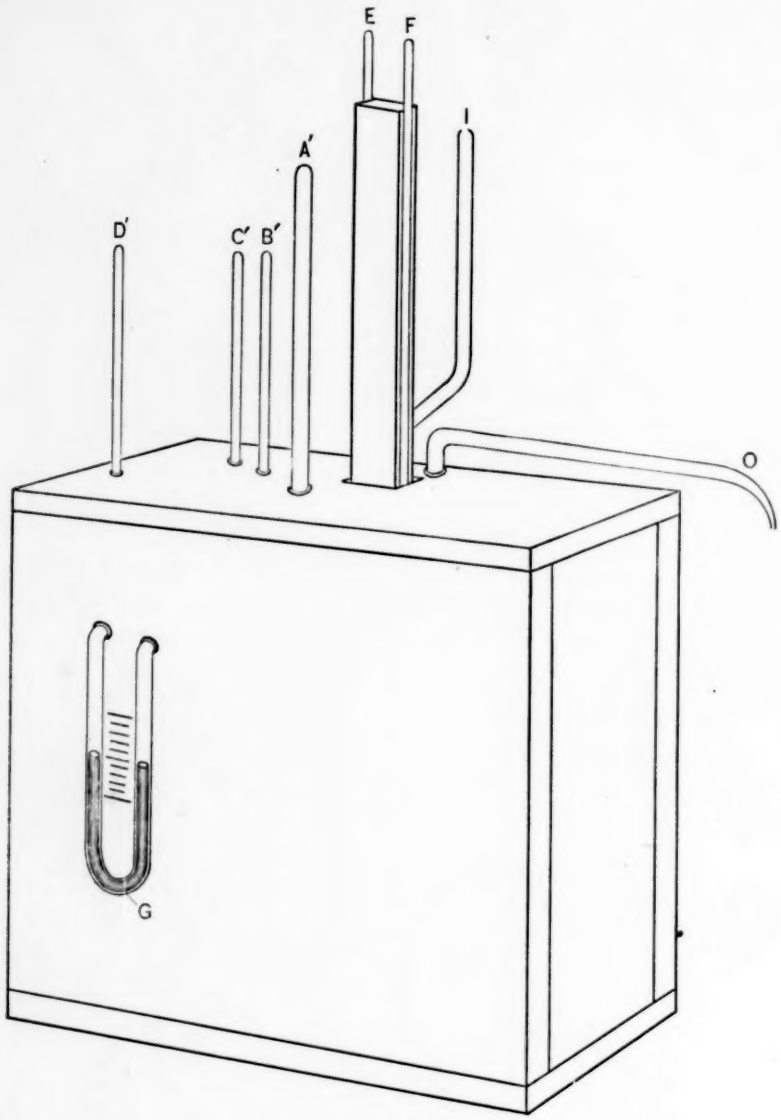

Fig. 1.

air chamber of this differential thermometer consists of a copper pipe about 4 meters long and 4 millimeters internal diameter, one coiled about and soldered to the lining, and the other coiled about and soldered to the jacket. One end of each pipe is closed and the

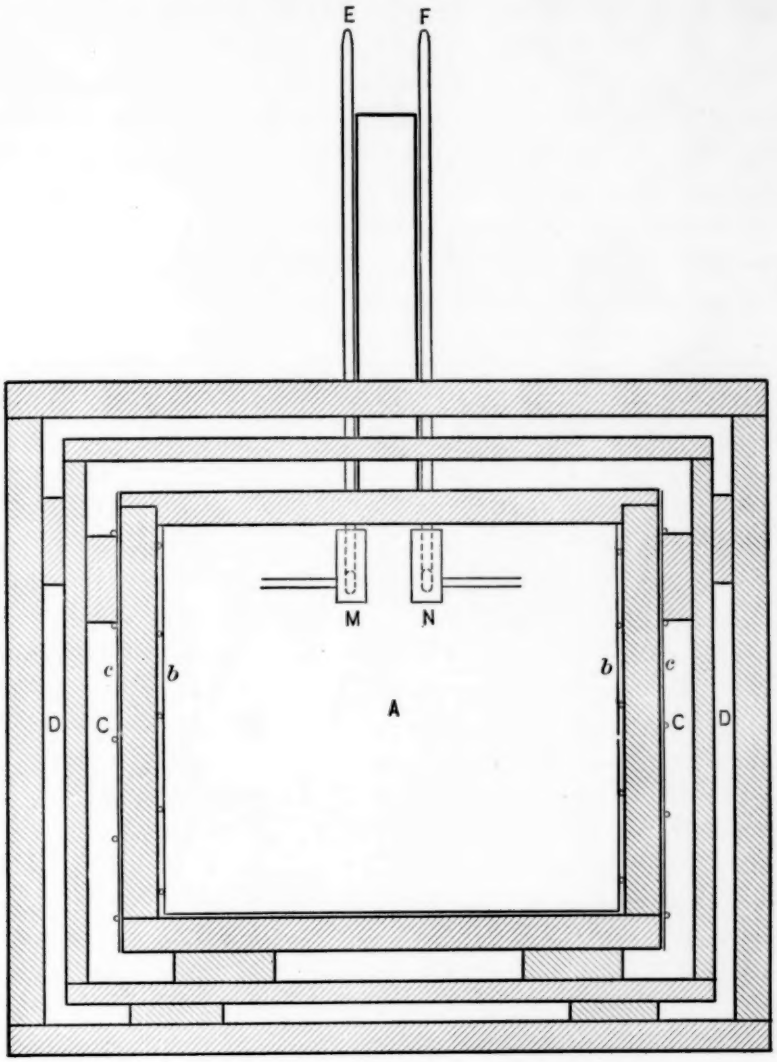

Fig. 2.

other connected to one end of the U-tube G, shown on the outside of the calorimeter in Fig. 1. The U-tube, which we call the gauge, contains kerosene oil, and serves to indicate any difference of temperature between the two copper walls. The zero mark is fixed after maintaining the whole calorimeter at a constant temperature for some hours. In order to keep the gauge reading sensibly zero,

and thus keep the two copper walls very closely at the same temperature, a coil of wire through which an electric current of any desired strength can be passed, is wound about the jacket, in the space *C*. And in order to make the regulation more perfect, a second coil is wound about the second box, in the space *D*, so as to maintain the temperature of this space nearly constant. The temperature of the chamber *A* is usually kept a little higher than the external temperature, so that no cooling is required, and by varying the currents in the two heating coils the temperature in *C* can be made to follow that in *A* so closely that the gauge readings are always small in their algebraic sum in any experiment zero. This eliminates all correction for radiation. In rare cases, when the temperature of the room has risen considerably, we have found it necessary to hang a wet cloth about the box to prevent the temperature of *D* rising above that of *C* and *A*. We intend to coil a small copper pipe in *D*, so that a stream of cool water may be sent through it, and then no difficulty will be encountered in the hottest weather.

In addition to the gauge, four thermometers, Fig. 1, indicate the temperatures of *A*, *b*, *c*, and *D*. That is, *A'* shows the temperature of the air in the calorimeter chamber *A*; *B'* has its bulb in a pocket of the lining *b*, and hence indicates the temperature of the wall *b*; *C'* similarly extends down into a pocket of the copper jacket *c*, and shows its temperature. Finally, *D'* gives the temperature of the outer air space *D*. *A'* is an accurate thermometer, reading from 20° to 25° C., graduated to 0°.01 and read to 0°.001 C. If *A'* shows the temperature to be constant during the whole period of an experiment, or the same for a considerable time near the end of an experiment that it was in the beginning, then there will be no correction for heat absorbed, or given up, by the apparatus. With both the "cooling correction" and the capacity correction eliminated, it remains to carry away and measure the entire heat generated by a condenser in *A*, or by any other source of heat within the calorimeter.

2. Carrying away and Measuring the Heat.

In order to carry away the heat generated a stream of water, which enters at *I*, Figs. 1 and 3, is made to flow through a coiled

copper pipe (Fig. 3) where it absorbs heat, and then leaves the calorimeter at *O*. In order to increase its absorbing capacity, the pipe is soldered to a sheet of copper, *LL*, both pipe and copper being painted black. Three such sheets, each with 4 meters of pipe attached, are joined together and placed side by side in the chamber

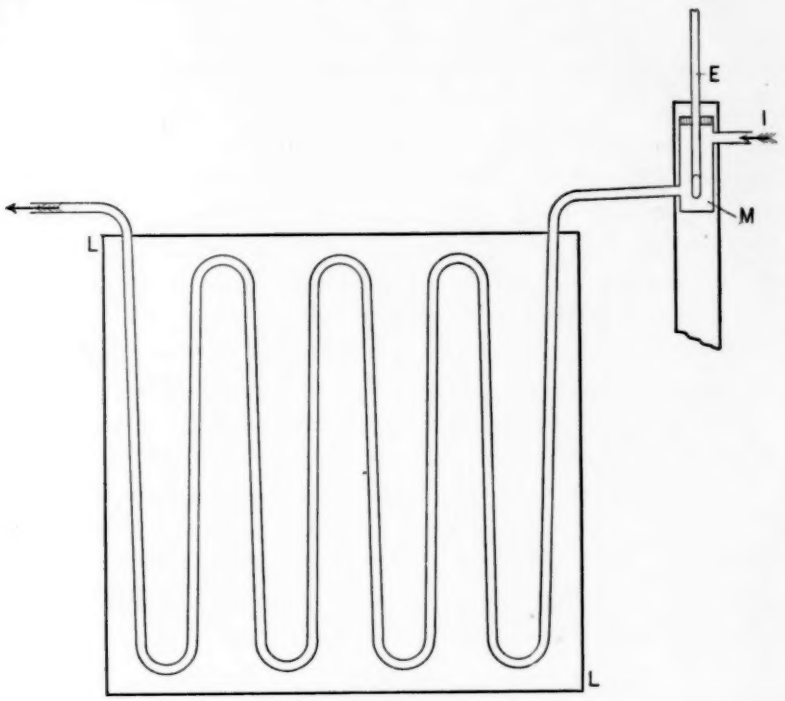

Fig. 3.

A, the condensers being slipped in between them. The rate of absorption of heat depends upon the difference of temperature between the absorbers and the air surrounding them. If a large amount of heat is to be brought away, the water is made to enter at a low temperature, and to flow rapidly through the absorbers. If a smaller quantity of heat is to be absorbed and carried away, the entering water will be warmer and its gain in temperature correspondingly less. By varying the temperature of the water and its rate of flow, the rate of absorption can be varied between wide limits, and kept very nicely at any desired point. In practice, the thermometer

A' is the guide in regulating the temperature of the entering water. If the temperature of A begins to rise (A', as already stated, can be read to one-thousandth of a degree) the entering water is slightly cooled; if to fall, it is slightly warmed; the rate of flow of water, after being once adjusted for a given experiment, is maintained constant.

In order to measure the quantity of heat thus carried away, the thermometers E and F are inserted in two small reservoirs, M and N, which stand in the wooden wall of the calorimeter, between the two copper surfaces. The thermometer E indicates the temperature of the water just as it enters the chamber A, and the thermometer F gives its temperature as it leaves. The difference of temperature multiplied by the mass of water per second gives the rate of absorption and removal of heat. The thermometers are accurately graduated and read to hundredths of a degree. The gain in temperature is several degrees, and may be ten or twenty degrees, by increasing the quantity of heat generated or reducing the rate of flow of water. Hence, the accuracy of the determination of the quantity of heat absorbed is sufficient for most purposes. The chief error is ordinarily due to changes in the temperature of the apparatus itself and its contents. By running the experiment several hours, however, and keeping it as nearly as possible at a constant temperature, this uncertainty is greatly reduced and the error made negligibly small.

The water flows into the calorimeter from a reservoir about a meter above, this height furnishing the necessary pressure. The temperature of the entering water is regulated by adding warm or cold water to the reservoir, and the rate of flow of the water is regulated by an adjustable valve. The water is collected in a liter flask, the time of each liter being recorded.

3. TEST OF THE CALORIMETER.

Table I. shows the results of one of the tests made upon the calorimeter. A current of electricity passed through a coil of wire within the chamber A, the electromotive force being measured by a carefully calibrated Weston voltmeter, and the current by a Kelvin balance. The experiment continued for a little more than four hours, while nine liters of water passed through the calorimeter.

TABLE I.

Test of the Calorimeter.

(*a*)	(*b*) Time.	(*c*) Period in Seconds for each 1,000 grams of Water.	(*d*) Average Temperature of the Ingoing Water.	(*e*) Average Temperature of the Outcoming Water.	(*f*) Increase in Temperature of each 1,000 grams of Water. (*e*)−(*d*.)	(*g*) Total Heat Measured (small calories). (*f*)×1000	(*h*) Small calories per Second. (*g*)÷(*c*)	(*i*) Equivalent Watts = *calories* × *J*. *h*)×4.1972
1	1 : 53 : 00							
2	2 : 19 : 45	1605	15.66	20.39	4.73	4730	2.947	12.37
3	2 : 46 : 50	1625	15.70	20.44	4.74	4740	2.917	12.24
4	3 : 13 : 55	1625	15.74	20.51	4.77	4770	2.935	12.32
5	3 : 41 : 15	1640	15.70	20.53	4.83	4830	2.945	12.36
6	4 : 09 : 10	1675	15.72	20.64	4.92	4920	2.937	12.33
7	4 : 37 : 10	1680	15.57	20.56	4.99	4990	2.970	12.47
8	5 : 04 : 10	1620	15.64	20.58	4.94	4940	3.050	12.79
9	5 : 32 : 15	1685	15.55	20.50	4.95	4950	2.938	12.33
10	6 : 01 : 00	1725	15.57	20.56	4.99	4990	2.893	12.14
								12.37

Electromotive Force = 20.0 volts.
Current = 0.617 amperes.
Watts (from electrical measurements) = 20.0 × 0.617 = 12.34.

The rate of absorption of heat was nearly, but not quite, constant, the temperature as indicated by A' having varied slightly. The final temperature was practically the same as that at the beginning, being slightly higher if anything. The average for the nine liters is 12.37 watts absorbed and carried away by the water, while the electrical measurements give 12.34 watts. By continuing the experiment longer and introducing greater refinements in the measurement of the current and electromotive force, a greater degree of accuracy could undoubtedly be attained. But this and other tests showed clearly that for our present purposes the calorimeter was abundantly accurate, and we proceeded to put some condensers into it and measure the heat evolved.

4. THE EXPERIMENTS.

Table II. gives the results of six experiments with the same beeswax and rosin condensers which were employed in our work by the

TABLE II.

Measurements of Energy Dissipated in Beeswax and Rosin Condensers.

(a) No. of the Condenser.	(b) Time.	(c) Period in Seconds for each 1,000 grams of Water.	(d) Average Temperature of the Ingoing Water.	(e) Average Temperature of the Outcoming Water.	(f) Increase in Temperature of each 1,000 grams of Water. (e)—(d.)	(g) Total Heat Measured (small calories). (f)×1000	(h) Small calories per Second. (g)÷(c)	(i) Equivalent Watts = *Average calories* × *f*. (h)×4.1972	(j) Frequency.	(k) Electromotive Force (volts). *E*	(l) Current (amperes) *I*	(m) Apparent Watts. *E*×*I*	(n) Cos ϕ (i)÷(m)	(o) Per cent. Loss = $\pi \cot \phi \times 100$. (n)×π×100	(p) Efficiency = $1 - \frac{100}{\pi \cot \phi}$. 100—(o)
(1)	9 : 36 : 00														
3, 4, 7.	10 : 02 : 00	1560	15.76	20.84	5.08	5080	3.256								
	10 : 25 : 00	1380	15.76	20.72	4.96	4960	3.595	13.34	140	868	.730	634	.0210	6.61	93.39
	10 : 51 : 30	1590	15.87	20.14	4.27	4270	2.686								
(2)	6 : 34 : 35														
1, 5.	7 : 12 : 30	2275	17.78	22.07	4.29	4290	1.886								
3, 4.	7 : 49 : 15	2205	17.57	22.08	4.51	4510	2.045								(30°)
8, 9.	8 : 26 : 30	2235	16.94	22.08	5.14	5140	2.300	9.36	26	1520	.40	608	.0154	4.84	95.16
	9 : 05 : 10	2320	16.59	21.97	5.38	5380	2.319								
	9 : 41 : 40	2190	16.23	21.92	5.69	5690	2.598								
(3)	2 : 56 : 30														
1, 5.	3 : 27 : 10	1840	17.62	21.60	3.98	3980	2.163								
3, 4,	3 : 55 : 45	1715	17.71	21.54	3.83	3830	2.233								(30°)
8, 9.	4 : 24 : 00	1695	17.61	21.48	3.87	3870	2.283	9.33	120	650	.897	583	.0160	5.06	94.94
	4 : 52 : 15	1695	17.66	21.45	3.79	3790	2.236								
	5 : 21 : 20	1745	17.58	21.43	3.85	3850	2.206								

(4)	7 : 00 : 40														(40°)
1, 5.	7 : 28 : 13	1653	17.65	21.58	3.93	3930	2.378								91.63
	7 : 56 : 40	1707	17.71	21.62	3.91	3910	2.291								
	8 : 25 : 47	1747	17.70	21.66	3.96	3960	2.267	9.54	140	805	.445	358	.0266	8.37	
	8 : 55 : 50	1803	17.68	21.69	4.01	4010	2.228								
	9 : 26 : 20	1830	17.63	21.66	4.03	4030	2.202								
(5)	2 : 38 : 30														(30°)
1, 5.	3 : 11 : 30	1980	20.64	22.65	2.01	2010	1.015								93.65
	3 : 44 : 20	1970	20.60	22.62	2.02	2020	1.025	4.25	137	630	.333	210	.0202	6.35	
	4 : 17 : 20	1980	20.56	22.58	2.02	2020	1.020								
	4 : 50 : 20	1980	20.68	22.64	1.96	1960	0.990								
(6)	8 : 37 : 20														(30°)
3, 4.	9 : 15 : 20	2280	20.55	22.27	1.72	1720	0.754	3.10	140	605	.333	202	.0154	4.82	95.18
	9 : 54 : 15	2335	20.59	22.28	1.69	1690	0.724								

Resonance Method. In each experiment a preliminary run not included in the table, allows the condensers and calorimeter to come to a constant temperature. Column *a* gives the number of the condensers in each case, all being joined in parallel to the same electromotive force. Column *b* gives the time of the beginning of each liter of water; column *c* the duration of each liter or 1,000 grams of water; column *d* the average temperature of the ingoing water as found from readings of thermometer *E*, taken regularly every five minutes, and column *e* the same for the outgoing water; *f* then shows the increase of temperature. Column *g* gives the number of calories of heat carried away by each 1,000 grams of water, and column *h*, which is the number in *g* divided by the corresponding number of seconds recorded in *c*, is the rate of absorption of heat. Column *i* gives the number of watts to which this is equivalent, taking *J*, the mechanical equivalent of heat to be 41,972,000 ergs. This is the value derived from Rowland's and Griffith's work, assuming the specific heat of water, at an average temperature of 20° C. to be unity. Column *j* gives the frequency. Sometimes this was estimated from the average frequency of the dynamo supplying the lines of the Middletown lighting circuits at the time, and in other cases it was determined by measuring the speed of a small synchronous motor. The electromotive force (*k*) was measured with an electrometer, the current (*l*) with a Siemens dynamometer. Column *n* gives the value of cos φ, of the expression Power = $EI \cos \varphi$. Care was taken in every instance to avoid the presence of upper harmonics, in some cases using a resonance coil to quench the harmonics as well as increase the voltage on the condenser. Column *o* gives the per cent. loss, $100\,\pi \cot \varphi$, and column *p* the net efficiency. This relative loss, $\pi \cot \varphi$, has been proved[1] to be the ratio of w to W, where w is the energy dissipated per half period, and W is the energy stored in the condenser at each charge. $1 - \pi \cot \varphi$, the net efficiency, is, therefore,

$$\frac{\text{Energy Stored} - \text{Energy Lost}}{\text{Energy Stored}}.$$

[1] See our paper, PHYS. REV.

5. Beeswax and Rosin Condensers.

The first experiment, with condensers Nos. 3, 4 and 7, showed a net efficiency of 93.39%, or a loss of 6.61%. The temperature of the condensers was not determined, but from the fact that the dielectric was softened and the quantity of heat generated was more than in any succeeding experiment, we feel sure that it was considerably above 40° C. Six condensers were then placed in the calorimeter, joined in three pairs. Nos. 1 and 5 gave no sound when joined to an alternating E. M. F. of 1,000 or more volts, and we called it the "best pair." In Nos. 3 and 4 vibrations were distinctly felt when the fingers were placed in contact with them, while the condensers gave a clear musical note, and on the higher voltages a hissing sound; this we called the "poorest pair." Nos. 8 and 9 were intermediate.

These six condensers were first all joined in parallel and connected to a low frequency circuit of 1,520 volts and 26 periods per second. Care was taken to exclude upper harmonics. The experiment continued over three hours after the temperature of calorimeter and condensers had become constant, by a preliminary run of several hours. The temperature of the calorimeter as indicated by the thermometer A' rose gradually for an hour, and hence the heat absorbed was less than the average. During the last hour the temperature was reduced, by quickening the rate of flow and cooling the entering water, so that the temperature was substantially the same at the end as at the beginning. The per cent. of loss is 4.84 at an average temperature of the condensers of 30° C. The voltage employed on this low frequency test was much higher than for any other experiment, and yet there was no evidence of brush discharge or appreciable leakage current.

In the third experiment the same condensers were subjected to a high frequency electromotive force, at 30° C., and the loss found to be 5.06%; that is, slightly greater than before. Hence, for a given voltage the energy dissipated *per period* would be slightly greater, and the energy dissipated *per second* more than five times as much.

In the fourth experiment only the "best pair" of condensers was used, and with a slightly higher voltage the temperature of the condensers rose to 40° C. Here the loss was found to be 8.37%,

nearly as much as the maximum value found by the resonance method.

The fifth experiment, with the "best pair," was made some days later, at 30° C., and the percentage loss came out 6.35%, that is, *greater* than the average of the six. This was unexpected, as well as the last result, which showed a loss for condensers 3 and 4, the "poorest pair," of 4.82%, which was *less* than the average. These results were then confirmed by an independent method, showing conclusively that the so-called "poorest pair" had the smallest loss; not, of course, *because* it emitted a distinct sound and hissed on high voltages, but in spite of that. The chief loss is doubtless due to some cause quite independent of the singing and hissing, and happens to be smaller where it would naturally be expected to be larger.

Thus we have confirmed by these calorimetric measurements the large values of the losses which we found by the resonance method in beeswax and rosin condensers, and also the existence of a well-marked maximum, as the temperature rises, beyond which the loss decreases considerably. It is an interesting fact that the residual charges of these condensers are very large, that they increase with the temperature up to 40° C. and then decrease as the temperature is carried higher. That is, *the maximum point for the residual charge is the same as for the energy loss.*

6. PARAFFINED PAPER CONDENSERS.

The second lot of condensers used were commercial paraffined paper condensers made by the Stanley Electric Company. A finished condenser is a solid slab about 25 x 30 cm. and 2 cm. thick. thus having a volume of 1,500 cc., and is enclosed in a tight tin case, the lead wires coming out through ebonite bushings. Nos. 1 to 4 of our condensers have a capacity of about 1.7 microfarad each; Nos. 5 to 10, which were purchased about a year later, have a capacity of about 3.2 m. f. each. The condensers of the second lot are made of paper about .0038 cm. thick, two sheets being placed together in each stratum. This we learned by dissecting some which we had broken down. It ought to be stated, however, that while the condensers are guaranteed by the makers to stand

500 volts alternating electromotive force, we have repeatedly subjected them to 1,000 to 2,000 volts, and in some cases for several hours at a time. Nos. 9 and 10 were upon one occasion maintained at 2,250 volts, at a frequency of 130, for over an hour, and showed no signs of being overtaxed. We have, however, broken several at voltages between 1,000 and 2,000. The paper of the first lot of condensers is thicker, but as we have never broken one of this lot we cannot state its thickness. From the fact that the capacity of each of these is about 60% as great as that of the others, while their volumes are substantially the same, we conclude that the thickness of the paper is about .0048 cm., supposing there are, as in the other, two sheets together in every stratum of the condensers.

In Table III. are given the results of seven separate experiments with Stanley condensers, which were made at intervals during the past three months. The frequency, in every case except experiment 4, was estimated from the average frequency of the two dynamos of the Middletown lighting circuits. Experiments 1 and 3 were made when the faster dynamo was supplying the lines; the others were with the slower dynamo. All but No. 4, however, were with a relatively high frequency. No. 4 was made using a two-pole rotary transformer, supplying it with direct current, and running it at a speed of 1,600 per minute. The percentage losses (o) vary more among the different condensers at the same frequency than one would expect.

The percentage loss at the frequency 28 (Experiment 4) is .78%, whereas at a frequency five times as great it is (Experiment 3) 1%. At 120 it is, as would be expected, nearly as great as at 140; that is, it is .96% (Experiment 2). Condenser No. 1 of the same lot shows a loss of 1.45% at frequency 140, which is 45% greater than Nos. 3 and 4 give. Of the second lot, No. 6 gives a large loss (Experiment 5) and other experiments which one of us has made by other methods show that all the other condensers of this lot have losses nearly the same as No. 6, excepting No. 10, which gives the smallest loss of any, .74% in one case and .78% in another (Experiments 6 and 7). Condenser No. 2 shows by other methods the same loss as 3 and 4. Hence we have the following singular results: *All of the first lot except one have a loss of 1% on high fre-*

TABLE III.

Measurements of Energy Dissipated in Stanley Paraffin Condensers.

(a) No. of the Condenser.	(b) Time.	(c) Period in Seconds for each 1,000 grams of Water.	(d) Average Temperature of the Ingoing Water.	(e) Average Temperature of the Outcoming Water.	(f) Increase in Temperature of each 1,000 grams of Water. (e)—(d)	(g) Total Heat Measured (small calories). (f)×1000	(h) Small calories per Secon (g)÷(c)	(i) Equivalent Watts = *Average calories* × *J*. (h)×4.1972	(j) Frequency.	(k) Electromotive Force (volts). *E*	(l) Current (amperes) *I*	(m) Apparent Watts. *E*×*I*	(n) Cos ϕ *i*÷(*m*)	(o) Per cent. Loss = $\pi \cot \phi \times 100$. (*n*)×π	(p) Efficiency = 1 — $\pi \cot \phi$. 1—(o)
(1)	7 : 24 : 00														
1.	7 : 59 : 50	2150	19.71	24.12	4.41	4410	2.051								
	8 : 36 : 30	2200	19.81	24.11	4.30	4300	1.955	8.36	140	1133	1.60	1813	.0046	1.45	98.55
	9 : 13 : 30	2220	19.82	24.20	4.38	4380	1.973								
(2)	12 : 33 : 25														
3, 4.	1 : 04 : 00	1835	18.06	23.30	5.24	5240	2.856								
	1 : 36 : 00	1920	18.04	23.43	5.39	5390	2.808								
	2 : 07 : 40	1900	17.93	23.40	5.47	5470	2.879	11.93	120	1264	3.1	3918	.0030	.96	99.04
	2 : 40 : 00	1940	17.98	23.47	5.49	5490	2.830								
(3)	7 : 17 : 20														
3, 4.	7 : 46 : 20	1740	16.71	22.09	5.38	5380	3.092								
	8 : 15 : 50	1770	16.59	22.12	5.53	5530	3.124	13.23	140	1194	3.5	4179	.0032	1.00	99.00
	8 : 45 : 23	1773	16.33	22.11	5.78	5780	3.260								
	9 : 14 : 43	1760	16.55	22.06	5.51	5510	3.131								

(4)	12 : 21 : 12														
3, 4.	1 : 21 : 25	3613[1]	18.45	22.46	4.01	2807[1]	0.777								
	2 : 37 : 30	4565	18.67	22.47	3.80	3800	0.832	3.47	28	1659	.837	1389	.0025	.78	99.22
	3 : 47 : 35	4205	18.80	22.46	3.66	3660	0.870								
(5)	12 : 24 : 20														
6.	12 : 55 : 10	1850	19.44	23.22	3.78	3780	2.140								
	1 : 26 : 45	1895	19.55	23.20	3.65	3650	1.926	8.34	120	778	2.34	1822	.0046	1.44	98.56
	1 : 59 : 30	1965	19.48	23.20	3.72	3720	1.893								
(6)	6 : 05 : 08														
10	6 : 41 : 30	2182	17.82	22.48	4.66	4660	2.136	9.17	120	1294	3.0	3882	.0024	.74	99.26
	6 : 54 : 30	780[2]	18.10	22.80	4.70	1740[2]	2.231								
(7)	2 : 17 : 55														
10	2 : 50 : 00	1925	17.96	22.35	4.39	4390	2.281								
	3 : 21 : 30	1890	18.05	22.39	4.34	4340	2.296	9.67	120	1294	3.0	3882	.0025	.78	99.22
	3 : 53 : 28	1918	17.98	22.44	4.46	4460	2.325								
	4 : 25 : 00	1952	17.92	22.44	4.52	4520	2.315								

[1] The amount of water for this period was 700 grams.

[2] The amount of water for this period was 370 grams.

quencies, and the exceptional condenser has a loss of 45% greater. All the condensers of the second lot have substantially the same losses, about 1.5%, and the exceptional one is scarcely more than one-half as much as the others; the exceptional one of the first lot having the same loss as all but one of the second. There is no possibility of a confusion of numbers, for they were plainly stamped when purchased and the capacities of the first and second lots are very different, as already stated. Our experiments do not indicate the reason for these large differences, but the existence of such differences is fully confirmed by measurements made by wholly independent methods, and which will shortly be published.

WESLEYAN UNIVERSITY, MIDDLETOWN, CONN., September 1, 1898.

A NEW TRANSFORMER DIAGRAM.

By Frank G. Baum.

THE following method for obtaining the instantaneous values of the primary and secondary currents and magnetic flux of a transformer is, so far as I am aware, new. The effort made was to express the instantaneous values of the three varying quantities—primary current, secondary current and magnetic flux—with their angular relations, in terms of the self-inductance and resistance of the two coils and the impressed e. m. f. The resulting diagram is simple and applies to every possible case; the method of obtaining it is straightforward. The diagram, as all other transformer diagrams, is valuable, perhaps, only in giving a mental picture of the whole phenomenon. The algebraic results, however, come out in such form that it is no more difficult to solve analytically any transformer problem than it is to solve any problem in the simple alternating current series circuit. In fact the solution of any transformer problem will be reduced to that of the latter.

In nearly all text-books treating of transformers the assumption is made at the outset, in order to simplify the problem, that the secondary current is 90° behind the primary current. This assumption, while nearly true when the transformer is carrying a small load, becomes less and less true as the external resistance is reduced, that is, as the load increases. This assumption I believe to be unnecessary. It also seems desirable to be able to explain all the changes introduced by all the varying elements from a single diagram. In the first part of this paper hysteresis and change of permeability will be neglected.

The following notation will be used:

b = instantaneous value of magnetic flux,
i_1 = instantaneous value of primary current,
i_2 = instantaneous value of secondary current,
E = maximum impressed e. m. f. on primary,

ω = 2π times the frequency,
R_1 = resistance of primary,
R_2 = resistance of secondary,
N_1 = number of turns on primary,
N_2 = number of turns on secondary,
L_1 = self-inductance of primary,
L_2 = self-inductance of secondary,
A = cross section of iron under induction,
l = length of magnetic circuit,
μ = permeability of the iron.

The three fundamental equations which follow need no explanation :

$$b = \frac{4\pi A\mu}{l}(N_1 i_1 + N_2 i_2) = K(N_1 i_1 + N_2 i_2), \tag{1}$$

$$E \sin \omega t = N_1 \frac{db}{dt} + R_1 i_1, \tag{2}$$

$$0 = N_2 \frac{db}{dt} + R_2 i_2. \tag{3}$$

From (1) we obtain, by taking the derivative with respect to t,

$$\frac{db}{dt} = K\left(\frac{N_1 di_1}{dt} + \frac{N_2 di_2}{dt}\right). \tag{4}$$

Multiply (2) by N_2 and (3) by N_1 and subtract, and we have

$$N_2 E \sin \omega t = N_2 R_1 i_1 - N_1 R_2 i_2,$$

which solving for i_1, gives us

$$i_1 = \frac{E \sin \omega t}{R_1} + \frac{N_1 R_2}{N_2 R_1} i_2. \tag{5}$$

Differentiating and dividing by dt we get

$$\frac{di_1}{dt} = \frac{E\omega \cos \omega t}{R_1} + \frac{N_1 R_2 di_2}{N_2 R_1 dt}. \tag{6}$$

Substitute for $\frac{db}{dt}$ in (1) its value from (4), and we have

$$E \sin \omega t = KN_1\left(\frac{N_1 di_1}{dt} + \frac{N_2 di_2}{dt}\right) + R_1 i_1. \tag{7}$$

From (6) and (7) we have, after collecting terms

$$\frac{KN_1}{N_2}\left(N_2^2+\frac{N_1^2R_2}{R_1}\right)\frac{di_2}{dt}+\frac{N_1}{N_2}R_2i_2=-\frac{KN_2^2E\omega\cos\omega t}{R_1}.$$

For convenience, write this

$$\frac{K_1di_2}{dt}+K_2i_2=K_3\sin\left(\omega t-\frac{\pi}{2}\right).$$

The solution of this linear equation is

$$i_2=\frac{K_3\sin\left(\omega t-\frac{\pi}{2}-\tan^{-1}\frac{K_1\omega}{K_2}\right)}{\sqrt{(K_2)^2+(K_1\omega)^2}}.$$

On substituting the values of K, K_1, K_2 and K_3 we get our final equation

$$i_2=\frac{E\left(\frac{M^1\omega}{R_1}\right)\sin\left[\omega t-\frac{\pi}{2}-\tan^{-1}\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)\right]}{R_2\sqrt{1+\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)^2}}.$$

Solution for i_1. Pursuing the same method as for the solution of i_2, we obtain the linear equation

$$K\left(N_1^2+\frac{N_2^2R_1}{R_2}\right)\frac{di_1}{dt}+R_1i_1$$
$$=E\sqrt{1+\left(\frac{L_2\omega}{R_2}\right)^2}\sin\left[\omega t+\tan^{-1}\left(\frac{L_2\omega}{R_2}\right)\right].$$

Solving this for i_1, we get

$$i_1=\frac{E\sqrt{1+\left(\frac{L_2\omega}{R_2}\right)^2}\sin\left[\omega t+\tan^{-1}\left(\frac{L_2\omega}{R_2}\right)-\tan^{-1}\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)\right]}{R_1\sqrt{1+\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)^2}}.$$

Solution for *b*.

From (3) we have $\frac{db}{dt}=-\frac{R_2i_2}{N_2}$;

[1] M = mutual induction of the two coils; $E\frac{M\omega}{R_1}=E\left(\frac{N_2}{N_1}\right)\left(\frac{L_1\omega}{R_1}\right)$.

which gives, on substituting for i_2 its value,

$$\frac{db}{dt} = -\frac{\frac{R_2 E}{N_2}\left(\frac{N_2}{N_1}\right)\left(\frac{L_1\omega}{R_1}\right)\sin\left[\omega t - \frac{\pi}{2} - \tan^{-1}\left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)\right]}{R_2\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}}.$$

Simplifying the right hand member and solving for b we have

$$b = \frac{4\pi A\mu N_1 E \sin\left[\omega t - \tan^{-1}\left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)\right]}{lR_1\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}}$$

If we take the maximum values of i_1, i_2 and b, we obtain the three equations :

$$I_1^2 R_1^2\left[1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2\right] = E^2\left[1 + \left(\frac{L_2\omega}{R_2}\right)^2\right],$$

$$I_2^2 R_2^2\left[1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2\right] = \left(\frac{E}{R_1}\right)^2 (M\omega)^2,$$

$$B^2\left[1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2\right] = E^2\left(\frac{4\pi\mu N_1}{l}\right)^2.$$

Instead of $1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2$ in these equations we may write $\sec^2\theta_{12}$; θ_{12} being the angle whose tangent is $\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}$. This will facilitate the calculation of the quantities i_1 and i_2 algebraically. The last equation may be written

$$H^2 l^2\left[1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2\right] = \left(\frac{E}{R_1}\right)^2 (4\pi N_1)^2$$

where H is the strength of field. In the latter form the equation shows the fall of magnetic potential. The equation shows the true reason for the drop in the secondary pressure as the load increases.

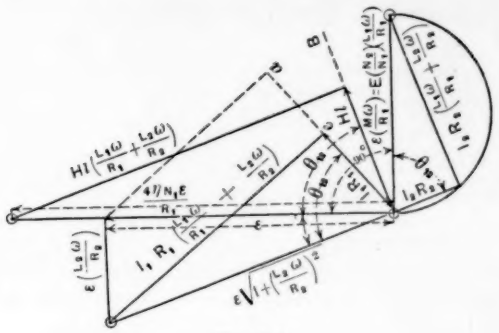

Fig. 1.[1]

Transformer Diagram. No hysteresis; non-inductive secondary.

$$I^2R_2^2\left[1+\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)^2\right]=E^2\left(\frac{M_1\omega}{R_1}\right)^2=E^2\left(\frac{N_2}{N_1}\right)^2\left(\frac{L_1\omega}{R_1}\right)^2$$

$$I_1^2R_1^2\left[1+\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)^2\right]=E^2\left[1+\left(\frac{L_2\omega}{R_2}\right)^2\right]$$

$$H^2l^2\left[1+\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)^2\right]=\left(\frac{4\pi N_1E_1}{R_1}\right)^2$$

$$\theta_{12}=\tan^{-1}\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)$$

$$\theta_2=\tan^{-1}\left(\frac{L_2\omega}{R_2}\right)$$

The diagram which follows (Fig. 1) will give a clearer idea of the relations of the quantities in the three equations. There appears to be an e. m. f. impressed on the primary having the value

$$E\sqrt{1+\left(\frac{L_2\omega}{R_2}\right)^2},$$

the phase being in advance of the real e. m. f. by an angle whose tangent is $\frac{L_2\omega}{R_2}$; the e. m. f. of self-induction is apparently

$$I_1R_1\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right).$$

[1] In Figs. 1, 1a, 1b and 2, for ε read E.

The e. m. f. impressed on the secondary has apparently the constant value of $\frac{EM\omega}{R_1}$, the phase being 90° behind the impressed pressure on the primary; the e. m. f. of self-induction is apparently

$$I_2R_2\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right).$$

We may, therefore, solve all problems in connection with the primary or secondary circuit without regard to the effects produced by the other coil. This fact renders easy, by this method, the solution of problems connected with the transformer when account is taken of all the varying elements—hysteresis included.

Neglecting, for the moment, hysteresis, let us see how the diagram may be employed to find the drop in effective pressure at the terminals of the secondary due to the increase of the angle of lag of the current in the secondary as the external resistance is decreased, that is, as the load is increased. Since $\frac{EM\omega}{R_1}$ is constant in direction and magnitude it is evident that the extremity of I_2R_2 will always lie on a circle on the former as a diameter. Lay off from the apparent e. m. f. of the secondary the angles

$$\tan^{-1}\left(\frac{L_1\omega}{R_1}\right)$$

(R_2 is large when the load is small, so that $\frac{L_2\omega}{R_2}$ may be neglected), and

$$\tan^{-1}\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right),$$

corresponding to a small and large load respectively, and drop perpendiculars on these lines from the extremity of the apparent pressure. The difference in length of the projected lines will give the fall in effective pressure due, as above stated, to the increase of the angle of lag of the secondary current. It will be noticed that in solving this problem the effect of the primary coil has not been directly considered (see Figs. 1a and 1b).

It is also evident that in all problems concerning the primary or

the line feeding it, *the transformer may be replaced by a single coil of self-inductance*

$$R_1\left(\frac{L_1}{R_1}+\frac{L_2}{R_2}\right),$$

and having the same resistance as the primary; the e. m. f. applied to the transformer terminals being replaced by one having the value

$$E\sqrt{1+\left(\frac{L_2\omega}{R_2}\right)^2}=E\sec\theta_2;$$

the phase being in advance of the true e. m. f. by the angle θ_2 (See Fig. 1*a*).

Making use of this fact it is possible to solve completely the problems of the multiple transformer circuit. Each transformer is replaced by a single coil as above and the current obtained for each transformer, the line current being then obtained by a geometric summation. This problem will be discussed at greater length later.

Effects of Varying Permeability.—The only change which a varying permeability will introduce will be to modify the form of the i_1, i_2 and b curves. To draw approximately the distorted curves we may assume H to vary as a sine function, and by getting values of μ corresponding to points on this curve the correction may be applied to the curves for i_1, i_2 and b. It has been shown, however, both practically and theoretically, that the wave form is not distorted appreciably by the change in μ.

The Effect of Hysteresis.[1]—The changes introduced by hysteresis can be more easily deduced if the phase of b be taken at ωt. If b and i_2 have the same maximum values as before, we may write

$$b=\frac{4\pi A\mu}{l}\frac{N_1E}{R_1}\frac{\sin\omega t}{\sqrt{1+\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)^2}}, \qquad (a)$$

and

$$i_2=\frac{E\left(\frac{M\omega}{R_1}\right)\sin\left(\omega t-\frac{\pi}{2}\right)}{R_2\sqrt{1+\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)^2}}, \qquad (b)$$

[1] See Bedell's Principles of the Transformer, for a graphical treatment of this subject.

since i_2 is always at right angles to b; its value we get from the equation

$$0 = \frac{N_2 db}{dt} + R_2 i_2, \quad \frac{db}{dt} \text{ being obtained from } (a).$$

Now, the maximum value of H is $\frac{B}{u}$, and, hence, if the angle between H and B be taken as φ (Figs. 2 and 3), we have, from (a)

$$hl = \frac{bl}{Au}\sin(\omega t + \varphi) = \frac{4\pi N_1 E \sin(\omega t + \varphi)}{R_1\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}}.$$

But hl (h is the instantaneous value of H) is the fall of magnetic potential, and this must be equal to 4π times the resultant ampère turns; this condition will determine the phase and magnitude of i_1. That is,

$$\frac{4\pi N_1 E \sin(\omega t + \varphi)}{R_1\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}} = 4\pi(N_1 i_1 + N_2 i_1).$$

Substituting in this the value of i_2 and solving for i_1, we get

$$i_1 = \frac{E\sqrt{\cos^2\varphi + \left(\sin\varphi + \frac{L_2\omega}{R_2}\right)^2}}{R_1\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}} \sin\left[\omega t + \tan^{-1}\frac{\left(\sin\varphi + \frac{L_2\omega}{R_2}\right)}{\cos\varphi}\right].$$

For I_1 a maximum this becomes

$$I_1^2 R_1^2\left[1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2\right] = E^2\left[\cos^2\varphi + \left(\sin\varphi + \frac{L_2\omega}{R_2}\right)^2\right].$$

The relations of the quantities in this equation are shown graphically in Fig. 3.

Consider first the case when the secondary is an open circuit (Fig. 2). If there were no hysteresis the conditions would be as shown by the full lines in the figure. Suppose H to be in advance of B by the angle φ. Then the phase of I_1 must coincide with H, and to

produce the same value of H as before, the value of the current must be the same, and, therefore, the value of i_1R_1, the e. m. f. necessary to overcome the resistance of the primary, must be the same as though there were no hysteresis. The only change, then,

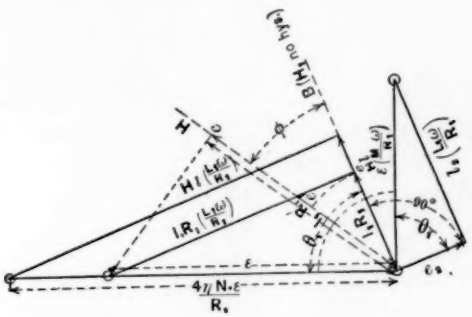

Fig. 2.

Secondary on open circuit;

$$\phi_1 = \tan^{-1}\left(\frac{L^1\omega}{R_1}\right);$$

ϕ, angle between H and B due to hysteresis.

is in the power delivered to the primary. The energy delivered is now $EI \cos(\theta_1 - \varphi)$, instead of $EI \cos\theta$, as would be the case if hysteresis were absent.

If the secondary is carrying a load the relations of the quantities which would exist if there were no hysteresis are shown in Fig. 1. Suppose that H is in advance of B as shown by the angle φ, Fig. 3. The equation for i_1 gives the phase and magnitude of I_1 to give the resultant field in the direction of H, the latter being of the same magnitude as before. The relations of the quantities when there is hysteresis are shown in Fig. 3. If $\varphi = 0$, that is, no hysteresis, Figs. 1 and 3 become identical.

The power delivered to the primary, with the secondary closed but no hysteresis assumed, is proportional to $(oa)(oe)$, Fig. 1; adding the effect of hysteresis the total power has a value porportional to $(ob)(oe'')$, Fig. 3.

Regulation.—The effects of the change of $\frac{L_2\omega}{R_2}$ on the regulation may be obtained from the diagram as before. It may be safely said

that hysteresis does not affect the regulation. Foucault currents will tend to demagnetize the iron as do the secondary ampère turns, but will not affect the regulation materially, as the demagnetizing effect of these currents is nearly constant. The pressure on the load will, of course, diminish as the current increases, due to the

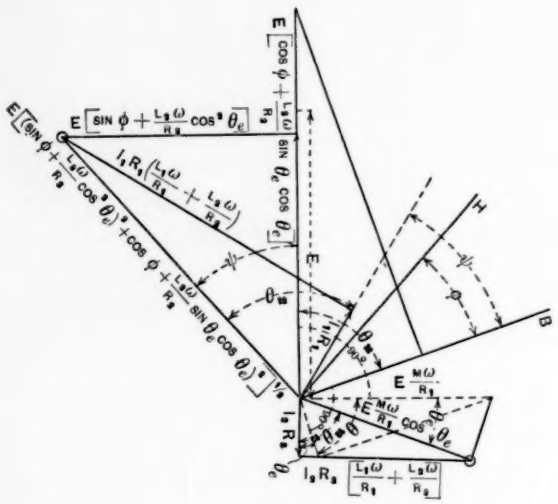

Fig. 3.

Transformer Diagram; seondary load. Non-inductive.

drop of pressure across the secondary resistance. The effect on the regulation of an increase of the resistance on the primary may be gotten from the secondary e. m. f. triangle. So long as the primary resistance remains constant the regulation is not affected. This is contrary to the statement usually made.

It is frequently assertedthat a transformer with no copper or iron losses would be self-regulating. This is seen to be in error from this or any other transformer diagram. The change of $\frac{L_2\omega}{R_2}$ which, as has been seen, affects the regulation, is independent of any internal resistance. We may, perhaps, conceive a transformer with no resistance, but we must have some secondary turns.

An Inductive Load.—Let B and H have the same values as before, that is, as defined by the equations

$$b = \frac{4\pi A u N_1 E \sin \omega t}{l R_1 \sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}}$$

and $$hl = \frac{4\pi N_1 E}{R_1} \frac{\sin(\omega t + \varphi)}{\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}},$$

(φ is the angle due to hysteresis).
Our equation of e. m. f.'s for the secondary now becomes

$$0 = \frac{N_2 db}{dt} + R_2 i_2 + L_e \frac{di_2}{dt},$$

in which Le is the coefficient of self-induction of the part of the circuit outside the transformer.

Putting in this equation the value of $\frac{db}{dt}$ and rearranging we get

$$\frac{Le di_2}{dt} + R_2 i_2 = -\frac{EM\omega}{R_1} \frac{\cos \omega t}{\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}}$$

$$= E\frac{M\omega}{R_1} \frac{\sin\left(\omega t - \frac{\pi}{2}\right)}{\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}}.$$

Solving for i_2, we obtain

$$i_2 = \frac{E\left(\frac{M\omega}{R_1}\right)}{R_2} \frac{\sin\left[\omega t - \frac{\pi}{2} - \tan^{-1}\left(\frac{Le\omega}{R_2}\right)\right]}{\sqrt{1 + \left(\frac{Le\omega}{R_2}\right)^2} \sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}}. \quad \text{(d)}$$

For i_2 a maximum, we have

$$I_2^2 R_2^2 \left[1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2\right] = \frac{E^2\left(\frac{M\omega}{R_1}\right)^2}{1 + \left(\frac{Le\omega}{R_2}\right)^2}$$

$$= E^2\left(\frac{M\omega}{R_1}\right)^2 \cos^2\theta_e,$$

where θ_e is the angle of lag due to the inductive load.

To determine I_1 we have

$$hl = \frac{4\pi N_1 E}{R_1} \frac{\sin(\omega t + \varphi)}{\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}} = 4\pi (N_1 i_1 + N_2 i_2);$$

which gives for i_1 the value

$$i_1 = \frac{E}{R_1\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}} \left[\sin(\omega t + \varphi) - \frac{\frac{L_2\omega}{R_2}\sin\left(\omega t - \frac{\pi}{2} - \tan^{-1}\frac{Le\omega}{R_2}\right)}{\sqrt{1 + \left(\frac{Le\omega}{R_2}\right)^2}}\right]. \quad (e)$$

Simplifying, we get

$$i_1 = \frac{E\sqrt{B^2 + C^2}\sin\left[\omega t + \tan^{-1}\left(\frac{B}{C}\right)\right]}{R_1\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}}, \quad (e)$$

in which

$$B = \sin\varphi + \tan\theta_2 \cos^2\theta e,$$
$$C = \cos\varphi + \tan\theta_2 \sin\theta e \cos\theta e.$$

Equations (e) and (d) will reduce to those previously obtained for a non-inductive load if θe be put equal to zero. For i_1 a maximum we obtain from (e)

$$I_1^2 R_1^2\left[1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R^2}\right)^2\right] = E^2(B^2 + C^2).$$

Fig. 4 shows graphically the relations of the quantities in equations (d) and (e). The diagram as drawn shows the decrease of effective pressure on the secondary, due to the inductive load, for the same impressed primary pressure; it also shows the decrease in power drawn from the primary. The secondary e. m. f. triangle shows very clearly the effect of an inductive load on regulation. For $\theta e = 0$, Fig. 4 reduces to Fig. 3.

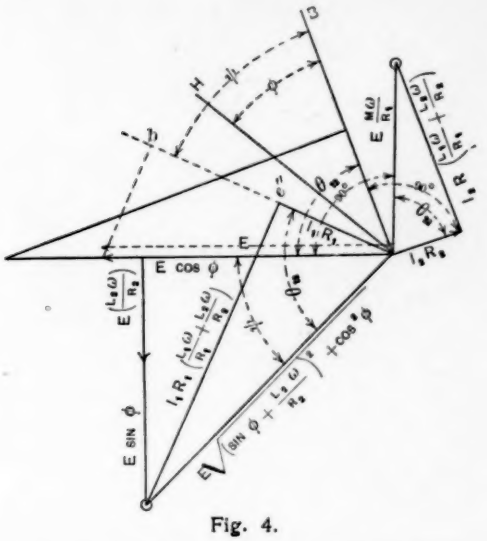

Fig. 4.

Transformer Diagram; inductive secondary load.

Capacity in the Secondary Circuit.—If instead of an inductive load we have a secondary with capacity, equations (*d*) and (*e*) reduce to

$$i_2 = \frac{E\frac{M\omega}{R_1}\sin\left(\omega t - \frac{\pi}{2} + \tan^{-1}\frac{1}{CeR_2\omega}\right)}{R_2\sqrt{1+\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)^2}\sqrt{1+\left(\frac{1}{CeR_2\omega}\right)^2}},$$

and

$$i_1 = \frac{E}{R_1}\,\frac{\sqrt{B^2+C^2}\sin\left(\omega t+\tan^{-1}\frac{B}{C}\right)}{\sqrt{1+\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)^2}},$$

A and B being defined by the equations above, θe being, however, equal to $\tan^{-1}\frac{1}{CeR_2\omega}$.

Inductance and Capacity in Secondary Circuit.—If both inductance and capacity are present in the secondary receiver circuit, we get

$$i_1 = \frac{E\sqrt{B^2 + C^2}\sin\left(\omega t + \tan^{-1}\frac{B}{C}\right)}{R_1\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}},$$

and

$$i_2 = \frac{E\frac{M\omega}{R_1}\sin\left[\omega t - \frac{\pi}{2} + \tan^{-1}\left(\frac{1}{CeR_2\omega} - \frac{Le\omega}{R_2}\right)\right]}{R_2\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}\sqrt{1 + \left(\frac{1}{CeR_2\omega} - \frac{Le\omega}{R_2}\right)^2}}.$$

In getting A and B, we must put

$$\theta e = \tan^{-1}\left(\frac{1}{CeR_2\omega} - \frac{Le\omega}{R_2}\right).$$

Two Secondaries in Parallel.—It might be of some interest to know to what form the equations reduce if we take the case of two secondaries wound on the same core.

We may at once write down the following equations:

$$E\sin\omega t = \frac{N_1 db}{dt} + R_1 i_1,$$

$$0 = \frac{N_2 db}{dt} + R_2 i_2,$$

$$0 = \frac{N_3 db}{dt} + R_3 i_3,$$

$$b = \frac{4\pi A\mu}{l}(N_1 i_1 + N_2 i_2 + N_3 i_3).$$

Solving[1] these i_1, i_2, i_3 and b, we obtain

$$i_2 = \frac{E\frac{M_{12}\omega}{R_1}\sin\left[\omega t - \frac{\pi}{2} - \tan^{-1}\left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2} + \frac{L_3\omega}{R_3}\right)\right]}{R_2\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2} + \frac{L_3\omega}{R_3}\right)^2}},$$

$$i_3 = \frac{E\frac{M_{13}\omega}{R_1}\sin\left[\omega t - \frac{\pi}{2} - \tan^{-1}\left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2} + \frac{L_3\omega}{R_3}\right)\right]}{R_3\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2} + \frac{L_3\omega}{R_3}\right)^2}},$$

[1] The method used is the same as that adopted in the first part of the paper. M_{12} is the mutual induction between coils one and two.

$$i_1 = \frac{E\sqrt{1 + \left(\frac{L_2\omega}{R_2} + \frac{L_3\omega}{R_3}\right)^2}\sin\left(\omega + \tan^{-1}\theta_{23} - \tan^{-1}\theta_{123}\right)}{R_1\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2} + \frac{L_3\omega}{R_3}\right)^2}}.$$

$$hl = \frac{4\pi N_1 E \sin\left[\omega t - \tan^{-1}\theta_{123}\right]}{R_1\sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2} + \frac{L_3\omega}{R_3}\right)^2}},$$

in which

$$M_{12} = \frac{4\pi A\mu N_1 N_2}{l},$$

$$M_{13} = \frac{4\pi A\mu N_1 N_3}{l},$$

$$\theta_{23} = \tan^{-1}\left(\frac{L_2\omega}{R_2} + \frac{L_3\omega}{R_3}\right),$$

and

$$\theta_{123} = \tan^{-1}\left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2} + \frac{L_3\omega}{R_3}\right).$$

If there are more than two independent secondary coils we will get, on solving for the currents in the different coils, expressions similar to the above. These may be written out at once. If hysteresis is taken account of, the expressions will take the forms of (*d*) and (*e*) previously obtained. These equations may be gotten by resorting to the devise used for the case of a single secondary coil, *i. e.*, by taking the phase of B as given by ωt. They may, however, be written out by inspection of the equations already obtained.

Transformers in Multiple.—It was stated in the first part of the paper that in the solution of any problem on the generator side of the transformer it (the transformer) may be replaced by a single coil having a coefficient of self-induction

$$R_1\left(\frac{L_1}{R_1} + \frac{L_2}{R_2}\right),$$

an equal resistance, the e. m. f. impressed on the coil being taken as

$$E\sqrt{1 + \left(\frac{L_2\omega}{R_2}\right)^2} = E\sec\theta_2.$$

This is on the assumption of no hysteresis and a non-inductive load. Equation (e), enables us to make a general statement applying to all cases. *Any transformer may be replaced by a single coil having a coefficient of self-induction equal to*

$$R_1\left(\frac{L_1}{R_1}+\frac{L_2}{R_2}\right),$$

an equal resistance, and an e. m. f. whose value is $\sqrt{B^2+C^2}$ *times the real e. m. f.; B and C being defined by the equations*

$$B = \sin\varphi + \tan\theta_2 \cos^2\theta e,$$
$$C = \cos\varphi + \tan\theta_2 \sin\theta e \cos\theta e,$$

in which φ *is the angle between H and B,*

$$\theta_2 = \tan^{-1}\frac{L_2\omega}{R_2}$$

and θe *is the angle of lag or advance—due to inductance or capacity, or both—of the secondary current from its normal phase at right angles to the magnetic flux. The apparent e. m. f. is in advance of the real e. m. f. by an angle whose tangent is* $\frac{B}{C}$. (See Fig. 4a.) In the same manner, for the solution of problems in the receiver circuit, *the transformer may be replaced by a single coil of self-inductance*

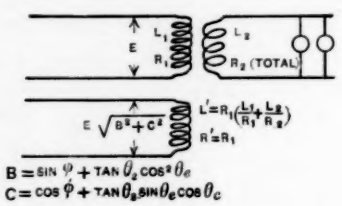

Fig. 4a.

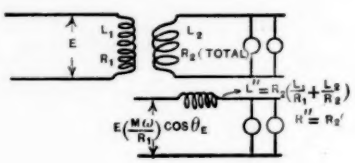

Fig. 4b.

Transformer replaced by single coil; inductive load. Fig. 4a for solution of problems in transmission circuit; Fig 4b for solution of problems in receiver circuit.

$$R_2\left(\frac{L_1}{R_1}+\frac{L_2}{R_2}\right),$$

an equal resistance, the e. m. f. at the terminals being taken as

$$\frac{EM\omega}{R_1}\cos\theta e\ ;$$

θ_e being the angle or lag or advance of the secondary due to a load containing inductance or capacity. (See Fig. 4b.)

In a multiple transformer circuit we may replace all transformers by single coils as above; we then have to solve the problem of a number of coils in parallel.[1] In case some of the transformers have two or more secondary coils we must use for $\tan \theta_2$ not $\frac{L_2\omega}{R_2}$ but

$$\frac{L_2\omega}{R_2} + \frac{L_3\omega}{R_3} + \cdots$$

Successive Transformation.—In long distance transmission of electric power at the present time the practice is to generate at a comparatively high voltage and to introduce step-up transformers before putting the voltage to line. At the other end of the line the voltage is again transformed down, usually once at a sub-station, and again at a point near the consumer of energy. In solving this problem we must start at the load end.

Replace each transformer in the multiple transformer circuit by an equivalent coil and find the total current taken by all the transformers (this may be done graphically). Replace all the parallel coils by a single coil, which will allow the same current to flow at the e. m. f. impressed on the primaries of the transformers. We now have the transformer at the step-down end of the line working on an inductive load. We can, next, replace this transformer and its load, including the resistance and reactance of the line, by a single coil (as shown on pages 12 and 13). Finally, we may replace the step-up transformer and its load by a single coil and get a solution for the current from the dynamo.

STANFORD UNIVERSITY, OCTOBER, 1898.

(A thesis presented for the degree of Electrical Engineer.)

[1] See Bedell and Crehore's Alternating Currents, p. 256.

INFLUENCE OF THE SURROUNDING DIELECTRIC ON THE CONDUCTIVITY OF COPPER WIRE.

BY JOSEPH FRANCIS MERRILL.

IN 1892 Professor Sanford, of Leland Stanford University,[1] published a paper describing some experiments that were undertaken with the object of finding if the dielectric surrounding a wire had any influence on its electrical conductivity. He tried the effect on a copper wire of about fifteen different dielectrics, liquids and vapors, and concluded that, in general, the conductivity was affected, in some cases increased and in others decreased, in one instance as much as 0.25%.

In a later paper on "A Necessary Modification of Ohm's Law,"[2] he referred to similar observations made upon the conductivity of a silver wire, but the effect was somewhat less than in the case of the copper wire and the same dielectrics.

An investigation under Professor Carhart's direction,[3] undertaken to confirm or refute Professor Sanford's conclusions, gave negative results for the effect of kerosene and alcohol, other dielectrics not being used.

But Sanford continued his investigations,[4] testing the effect of various dielectrics on five different copper wires varying in diameter from 0.35 mm. to 1.66 mm. He found effects with a copper wire 1.27 mm. diameter, but on taking another piece from the same spool and hammering it flat no change of resistance due to the dielectric was observed. He did not find any definite relation between the amount of surface exposed and the effect due to the dielectric. The amount of the effect of any dielectric seemed to depend on the particular wire used, since the percentage effect on no two wires was the same.

[1] Leland Stanford, Jr., Univ. Publications, Studies in Electricity, No. 1.
[2] Phil. Mag , vol. 35, p. 65, 1893.
[3] PHYSICAL REVIEW, vol. 1, p. 321, 1894.
[4] PHYSICAL REVIEW, vol. 3, p. 161, 1895.

The conflicting results obtained by Sanford and Carhart led Drs. Grimaldi and Platania, of the University of Catania, to take up the investigation with a different method; and in their memoir[1] they detail the observations they made on a copper wire 30.4 cm. long and 0.22 mm. diameter, using air and petroleum as dielectrics. They found that petroleum had an effect, decreasing the resistance nearly 0.015%, which is only about one-twelfth the percentage effect observed by Sanford.

In view of these different results it appeared to me that a "necessary modification of Ohm's law" had not been established. And it seemed worth while to test the matter further on account of the light that might possibly be thrown on the nature of metallic conductivity. But the method employed by Sanford and by Carhart was open to the objection that it was an absolute and not a comparative one. Furthermore, the method of measuring temperature did not appear as refined as the minuteness of the suspected phenomenon demands. It therefore occurred to me that if the length of wire immersed in the dielectric could be increased, and its temperature more accurately measured, the comparative method being employed to determine both resistance and temperature differences, that the phenomenon could, beyond question, be proved to exist, provided it was of the order of magnitude reported by Sanford.

Accordingly in March, 1897, with the very helpful suggestions of Professors Rowland and Ames, I began to devise a comparative method which enabled me to prove that if the various dielectrics used changed the resistance of the wires employed, this change was in no case greater than 0.01%; or there was no change greater than the probable limits of experimental error. And with slight modifications the method is capable of giving more accurate results than I obtained.

At the time of devising this method, I was entirely ignorant of the fact that Drs. Grimaldi and Platania had already employed a comparative method which, as I later learned, the one used closely resembled. But it was nearly a year after I completed my obser-

[1] Sulla Resistenza Elettrica dei Metalli nei diversa Dielettrici, Parte I., Ricerche sulla variazione di resistenza del rame nel petrolio, 1895. Also, Nuovo Cimento, July, 1895.

vations when, through the kindness of Professor Carhart, I first had access to their memoir and learned what method they had employed; so that I was not in the slightest degree indebted to them for suggestions.

Figs. 1, 2 and 3 will indicate the method made use of. Fig. 1 shows the arrangement at the top of the tubes, and Fig. 2 that at

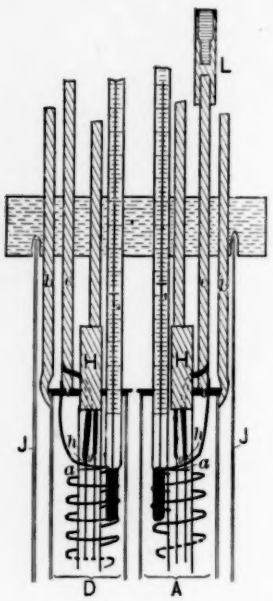

Fig. 1.

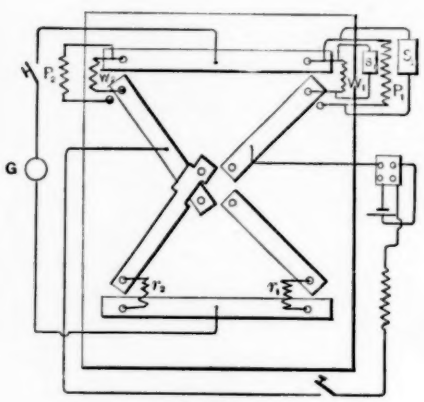

Fig. 2.

the bottom. Two copper tubes *A*, *D*, 100.7 cm. long and 3.2 cm. diameter, were each provided with a mercury thermometer graduated to tenths of degrees, a platinum thermometer, and a coil of polished copper wire. These tubes were then placed side by side within a double-walled tubular water-jacket made of zinc. One tube contained air only and the resistances of its copper coil and platinum wire were each to serve as a standard with which was to be compared the resistance of the corresponding wire in the other tube, which was to be filled in turn with various dielectrics.

These tubes were side by side in an air column, which was in turn surrounded by a water column 8.5 cm. thick and 120 cm. high. The inner tube of the water jacket enclosing the air column was

8.4 cm. diameter and terminated at the bottom in a funnel, the stem of which, 8 cm. long and 1.2 cm. diameter, projected 1 cm. below the flat bottom of the outer jacket, which was 25.4 cm. diameter and 122 cm. high. Up through this stem and funnel passed a thick-walled rubber tube which connected with the inside of the dielectric tube, and through which liquids and vapors could be run in or out at will without in any way disturbing either of the copper tubes. When the dielectric tube was filled, the rubber tube was closed at the bottom by a pinchcock.

The mercury thermometer bulbs extended about 6 cm. below the top of the tubes, and the thermometers served as indicators of the approximate temperature of the interior of the tubes. The readings from these were not used in deducing results. Each platinum thermometer was made of 185 cm. of wire 0.25 mm. diameter, doubled on itself and secured within a thin-walled copper tube 9 mm. diameter and about 99 cm. long, which was fastened so as to pass lengthwise through the center of the large copper tube, but was insulated for it at each end by vulcanite. The thermometer tube was closed liquid-tight at the top by a copper plug *H*, and at the bottom by a brass plug *p*. One end of the platinum wire was secured at *g* to a short piece of copper wire, the other end of which was soldered to *H*. The other terminal of the platinum wire was fastened at *h* to an insulated copper wire which passed out through a small hole in *H* and was then soldered to the copper terminal *e*. The two portions of the platinum wire were insulated from each other and from the containing tube by thin "fiber" discs *a*, placed at suitable intervals throughout the length of the tube. At the bottom where the wire was doubled on itself a fastening to the plug *p* was made by means of a piece of "fiber" *f* and a wire *s*. Thus, extending from top to bottom, the platinum thermometer would indicate the *average* temperature of the tube in which it was placed.

The helix was made from about 39 feet of polished copper wire 1 mm. thick and was 1.9 cms. diameter. One end was soldered to the copper cap closing the bottom of the tube and the other, after passing through the vulcanite plug at the top, to the terminal rod *e*. Electrical contact with the thermometer tube was prevented by four

long thin glass rods, tied 90° apart to the tube. The two helices were made from the same spool of wire and were almost of the same length, that of the air tube being slightly greater. The platinum resistance in the air tube was also slightly greater than that of the dielectric tube.

Tubes *A* and *D* were alike in all respects and each tube and helix, or its platinum thermometer, formed one arm of the bridge, while the other tube formed another. Connection was made with the bridge by means of heavy flexible cords terminating in rods which were held firmly to the bottom of the bridge mercury cups by elastic bands that were fastened one end to the bridge and the other to hooks in the terminal rods. Each rod was held by two bands. Each flexible cord was composed of 110 strands of No. 24 copper wire. These cords were attached to the tube terminals in the way indicated at *L*, Fig. 1. *L* was a copper rod, 7.3 mm. diameter, bored out at each end, the terminal *e* tightly fitting the lower hole and the upper receiving the snugly-fitting cord-terminal, which was then securely fastened by allowing to solidify the Griffith fusible metal with which the hole was previously partly filled. This metal fused at such a low temperature that the connections could be broken without difficulty.

The copper rods *b* and *e*, each 4 mm. diameter, were then the terminals of the helix and tube *A* (or *D*), but *e*, being also a platinum terminal, the copper and platinum resistances had one common lead from the tube to the bridge. Hence to change from copper to platinum resistances but one terminal rod on each side of the bridge had to be changed, together with the spool on which was wound the two fixed arms.

The fixed arms for the copper resistances were wound non-inductively on the same spool—the wires all being wound together so that subsequently there would be no difference in temperature—and differed in resistance by about 0.0001 ohm, the resistance of each being about 0.3075 ohm at 27° C.—but little greater than that of the copper tube and helix. The fixed arms for the platinum resistances were similarly made of No. 25 German silver wire. These arms differed in resistance by 0.0003 ohm, each being about 4.6 ohms.

Fig. 3 makes the bridge connections plain. W_1 and P_1 represent

the copper and platinum resistances, respectively, of the air tube A, W_2 and P_2 the corresponding resistances of the dielectric tube D, r_1 and r_2 the fixed arms of the bridge. The bridge was balanced by shunting W_1 or P_1 with resistance from the standard plug box S_1 or S_2, respectively. If the two resistances to be compared are nearly equal and the divisions in the shunt box small enough, this is a zero method even with the most sensitive galvanometer. It is the simplest and best method with which I am acquainted for comparing two nearly equal resistances one of which is known, and was suggested to me by Professor Rowland. But the smallest divisions in S_1 being units, I determined fractions by interpolation from galvanometer deflections. The electric current was furnished by one storage cell, and a fixed resistance of 106 ohms was included in the battery circuit so as to cut down the total current to 20 milliampères or less.

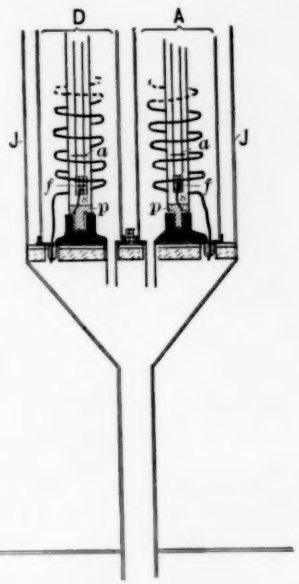

Fig. 3.

The galvanometer was a four-coil instrument made in the Physical Laboratory of this University. The coils were all connected in parallel their resistances being then about 3.4 ohms. The galvanometer was furnished with a Wiess system, the period of which was made about 14 seconds. Deflections were observed by means of a telescope and scale with mm. divisions, the scale being about 110 cm. from the galvanometer mirror.

To prevent sudden changes of temperature in any part of the connections, the water jacket was thickly wrapped in felt, and the bridge, shunt boxes, keys, tube terminals, and galvanometer were enclosed in boxes and cases. The top of the water jacket was closed by two board covers on which the box enclosing the tube terminals rested. The keys were operated by push rods and levers from the outside of the boxes. The box containing the shunt resistances had a double top; the inner one, just above the shunt boxes,

was pierced with small suitably placed holes through which the plugs passed, and the other one was made of strips only one of which was removed to change the plugs. Thus the heat of the hand did not reach the shunts. The galvanometer circuit was kept closed continuously, except during the instant of closing and opening the battery circuit, to reduce to a minimum the disturbance due to thermal currents. The bridge terminals and the commutator could be changed from the outside of the enclosing box. When the terminals of one set of resistances were removed from the bridge to allow those of the other set to be connected, they were put into adjacent mercury cups to prevent contamination of the amalgamated surfaces.

Tests at intervals showed that tubes *A* and *D* were satisfactorily insulated from each other and from the jacket. This was accomplished by wood soaked in paraffin and "fiber." The tops of these tubes were about 6 cm. below the level of the surrounding water and the under surface of the wood cover of the jacket.

With the arrangement indicated above it was thought that a differential method was obtained which would give good results. The conductivity of a copper wire in the air tube was to serve as a standard with which the conductivity of a wire in the dielectric tube was to be compared, the two wires being just alike in material and arrangement and of almost equal resistance. In order to measure the temperature differences of these wires, the platinum thermometers were to be employed and these, extending through the entire length of the tubes, would give the average temperature.

After the platinum thermometers were made the wires were annealed, and a careful determination of the temperature coefficient of each gave 0.00180. The coefficient of the copper tube and helix was 0.0037. After the tubes had stood in place several hours surrounded by a nearly constant temperature water bath, measurements were made by means of a Queen's standard bridge, Anthony form, of the absolute resistance of the air tube and helix, and its platinum thermometer. The measurements gave for the tube and coil 0.27576 ohm, and for the platinum 7.3884 ohms at 22°.7 C. Observations were then taken at intervals during four days, water from the mains of nearly constant temperature running through the

jacket day and night, to determine the difference between the resistances of the two tubes and their coils. These observations were made on the 6th, 7th, 8th, and 9th of Aug. The values for the second tube, air being the dielectric, were found to be 0.274279 ohm and 7.37513 ohms for copper and platinum, respectively. These latter values were obtained, not by use of the Queen's bridge, but by connection with the comparison bridge.

The order of measurement was to secure a balance with copper resistances by varying the shunt resistance of the copper tube and helix used as standard, the commutator being in one position and the current in one direction; then, with commutator as before, reverse the current and observe the shunt resistance. Next the commutator was reversed and a balance secured with both direct and reverse current. The commutator was then turned back to the first position and the whole series of measurements repeated. Following this, the platinum thermometer terminals were attached to the bridge and the same series of measurements made. Next the copper tube terminals were put back and the measurements repeated. But nearly always direct and reverse currents would give the same values so that measurements with reverse currents were not always made.

If $P_1 \equiv$ resistance of the platinum thermometer of air tube,
$P_2 \equiv$ resistance of the platinum thermometer of dielectric tube,
$W_1 \equiv$ the resistance of the copper air tube and helix,
$W_2 \equiv$ the resistance of the copper dielectric tube and helix,
$R' \equiv$ the resistance of the shunt when commutator is in one position,
$R'' \equiv$ the resistance of the shunt when commutator is in other position; then

$$W_2 = \sqrt{\frac{W_1 \times R'}{W_1 + R'} \times \frac{W_1 \times R''}{W_1 + R''}} = \frac{\frac{W_1 \times R'}{W_1 + R'} + \frac{W_1 \times R''}{W_1 + R''}}{2} = \frac{R_1 + R_2}{2}$$

and

$$P_2 = \sqrt{\frac{P_1 \times R'}{P_1 + R'} \times \frac{P_1 \times R''}{P_1 + R''}} = \frac{\frac{P_1 \times R'}{P_1 + R'} + \frac{P_1 \times R''}{P_1 + R''}}{2} = \frac{R_1 + R_2}{2}$$

where, in this second case, R' and R'', R_1 and R_2 are, of course, not the same values as they are in the first case.

DIELECTIC KEROSENE.

Date	Hour	T	T_1	T_2	R'	R''	R
Aug.	P. M.				Cu.		
9	1:45	24.7	22.83		51.15	49.34	
					Pt.		
	2:30	25.0	23.00		4130	3785	
					4120	3775	
					Cu.		
	4:30	25.3	23.04		51.21	49.50	
					Pt.		
	4:40	25.3	23.04		4210	3855	
					4205	3855	
					4207	3855	7.37485
					Cu.		(A)
	5:10	25.4	23.04		51.21	49.60	
					51.21	49.55	0.274261
Aug.	A. M.				Pt.		
10	10:55	24.4	22.59	22.59	4255	3880	
					4250	3875	
					4240	3880	
					4248	3878	7.37495
					Cu.		
	11:15	24.4	22.60	22.60	51.70	49.29	
					51.32	49.33	(B)
					51.70	49.50	
					51.55	49.45	
					51.56	49.40	0.274264
	P. M.				Cu.		
	3:10	24.4	22.65	22.65	51.90	49.78	
					51.63	49.38	
					51.80	49.66	
					51.80	49.60	0.274268
					Pt.		
	3:45	24.4	22.65	22.65	4260	3890	(C)
					4260	3890	
					4260	3885	
					4260	3888	7.37500
Aug.	A. M.				Cu.		
11	8:00	23.6	22.30	22.30	52.45	50.33	
					52.26	50.24	
					52.00	50.00	
					51.90	50.10	
					51.85	50.10	
					52.10	50.15	(1)

DIELECTRIC KEROSENE.—(*Continued.*)

Date.	Hour.	T	T_1	T_2	K'	K''	K
					Pt.		
Aug. 11	A. M. 8:40	23.7	22:35	22.35	4325	3945	
					4330	3940	
					4325	3935	
					4320	3930	
					4315	3935	
					4322	3937	(a)
					Pt.		
	10:10	23.8	22:40	22.40	4340	3950	
					4335	3945	
					4337	3947	(b)
					Cu.		
	10:20				51.85	50.5	
					51.85	50.3	
					51.90	50.5	
					51.70	50.2	
					51.90	50.3	
					51.84	50.36	(2)
					Cu.		
	11:00	23.8	22:40	22.40	52.4	50.85	
					52.4	50.75	
					52.4	50.80	(3)
					Pt.		
	11:05				4370	3975	
					4360	3970	
					4360	3965	
					4363	3970	(c)
					Cu.		
	11:18				52.3	50.48	
					51.9	49.65	
					52.0	50.10	
					52.6	49.70	
					52.0	50.00	
					52.16	50.00	(4)
					Cu.		
	P. M. 3:35	23.8	22.40	22.40	52.00	49.65	
					52.25	50.20	
					52.50	50.55	
					52.60	50.25	
					52.30	49.90	
					52.33	50.11	(5)

DIELECTRIC KEROSENE.—(*Continued.*)

Date.	Hour.	T	T_1	T_2	R'	R''	R
Aug.	P. M.				Pt.		
11	3:55	23.8	22.40	22.40	4325	3950	
					4310	3945	
					4300	3930	
					4300	3925	
					4300	3920	
						3920	
					4307	3931	(d)
					Cu.		
	4:15	23.9	22.40	22.40	51.40	49.85	
					51.50	49.50	
					51.60	49.80	
					52.00	49.40	
					51.75	49.75	
					51.65	49.66	(6)

The observations on the morning of August 9th, giving nearly identical results for P_2 and W_2 as those made on the morning of the 8th, it was thought that the tubes had become of the same temperature, especially since they had been standing undisturbed for four days. Hence at 11 o'clock August 9th the dielectric tube was filled with kerosene by running the liquid in through the bottom of the tube as previously described, none of the connections or apparatus being in the least disturbed. When first run in the kerosene lowered the temperature of the dielectric tube 0°.3 C. below that of the air tube, according to the mercury thermometers. After standing some time so as to allow temperature conditions to become stable I began to record the observations made. I reproduce them all here just as they were written at the time, together with the subsequent deductions.

$$\left.\begin{array}{ll} P_1 = 7.3884 \text{ ohms} & P_2 = 7.37513 \\ W_1 = 0.27576 & W_2 = 0.274279 \end{array}\right\} \text{ohms in air.}$$

$T \equiv$ temperature of interior of box enclosing the bridge and shunt boxes.

$T_1 \equiv$ temperature near top of air tube.

$T_2 \equiv$ temperature near top of dielectric tube.

$R \equiv$ the resistance of dielectric tube and coil, or its platinum thermometer, as the case may be, *i. e.*, W_2 or P_2. T, T_1, and T_2 are readings made on mercury thermometers but these observations were not used in deducing results.

From set of observations (A) the platinum resistance in the dielectric tube is 0.00028 ohm less than the air value, corresponding to 0°.021 C. The copper resistance is 0.274261 ohm, but corrected for 0°.021 C. it is 0.274282. Correcting (B) and (C) in the same way for temperature differences of 0°.015 and 0°.010, respectively, according to the platinum thermometer indications, the value of W_2 is 0.274279 and 0.274278 respectively—the exact air value.

The mean of (1, 2, 3, 4, 5, 6) gives $R' = 52.1$ and $R'' = 50.2$, from which is obtained 0.2742805 for W_2.

Mean of (a, b, c, d) gives $R' = 4332$ and $R'' = 3946$, from which is obtained for P_2 7.37520 ohms.

Now these values are almost identical with those obtained with air in the dielectric tube, the copper being 0.0000015 ohm greater and the platinum 0.00007 ohms greater, which corresponds to 0°.005 C. This would give for copper, corrected, 0.274285, an increase of 0.002 per cent., according to these observations.

Now, for a copper connection a change of one ohm in R' corresponds to a possible resistance change of about 0.000025 ohm in the dielectric tube and helix. But a change of 1 ohm caused a galvanometer deflection of 22 scale divisions; hence a deflection of 1 scale division would be caused by a resistance variation of one-millionth of an hour. With the platinum where $R' \equiv 4350$ ohms, a change of 5 ohms would correspond to 0.00002 ohm, *i. e.*, to about 0°.0016 C.

Sanford found a *decrease* in resistance of kerosene of 0.18 per cent., which would correspond in the tube and wire here used to 0.00049 ohm. The sensibility of the galvanometer during these observations was such that it seems if there had been a change of even 0.00001 ohm, or one-fiftieth of Sanford's percentage change it could have been detected.

The variations in the value of the shunt resistances were undoubtedly mostly due to variable commutator and terminal contacts. This certainly was the case so far as variations in the same group of measurements are concerned.

The introduction of the kerosene lowered the temperature of the dielectric tube about 0°.3 below that of the air tube. It is hardly probable, therefore, that it subsequently became higher. According to the platinum thermometers it was always slightly lower. But notwithstanding this the resistance of the dielectric tube, according to measurements made even three hours after the introduction of the kerosene, was only 0.007 % less than its air value; and this difference disappeared when temperature corrections were applied. Thus temperature and the reported effect of kerosene here acted in the same direction. It is remarkable, therefore, that this difference should be no greater than 0.007 %, since kerosene alone produced, for others, effects varying from 0.015 % to 0.18 %. The only conclusion is that kerosene here produced no effect.

After the above measurements the kerosene was run out of the tube and the latter dried by running gasoline in and out, and then blowing and drawing air through by a foot bellows and suction pump, respectively. But a momentary overflow of the water jacket necessitated the removal of the tubes for drying before another set of observations with air dielectric could be made; some of the connections having been broken no values comparable with the previous air values could be obtained.

Some other dielectrics were then tested in the same way as kerosene had been. The first after kerosene was a sample of absolute alcohol bought as pure. But this alcohol produced an unexpected result. Owing to contact e. m. f. it was impossible to take readings with the platinum thermometer; for the galvanometer needle would be violently deflected on closing the galvanometer circuit when the platinum terminals were attached to the bridge. And with the copper resistances connected to the bridge there was a deflection of 2–3 cm. produced by closing the battery circuit, this deflection being exactly the same as would be caused by self-induction in one arm of the bridge. (No such effects were observed with kerosene, benzine and other true dielectrics.) But after standing fifteen hours the temperature of the tubes, according to the mercury thermometers, had become the same, and the observations then made resulted in giving W_2 a value only — 0.001% less than its air value.

Running the alcohol out and testing for the effect of its vapor

(the platinum thermometers now being used), there was an increase in W_2 of 0.01% as compared with the air value obtained before the alcohol was run in, but an effect of — 0.001% when compared with the air value obtained after the vapor was pumped out. That the alchohol had acted chemically upon the wire, permanently increasing its resistance, seemed beyond doubt: however, to make sure of this another portion of the same alcohol was again tested, but with the same results as before. Assuming the temperature of the two tubes to be the same after standing all night, the resistance in the alcohol was slightly less than in air, while in the alcohol vapor it was 0.009% greater, compared with the previously obtained air value, but identically the same compared with the air value obtained after the vapor was pumped out. The specific resistance of this alcohol was subsequently found to be about 30,000 ohms; of kerosene it was infinite.

The purest methyl alcohol obtainable in Chicago was then tried, but it was so strongly electrolytic that no measurements could be made with it. Its specific resistance was 7,500 ohms.

After making two separate tests of the effect of the sulphuric ether vapor the tubes were removed from the jacket and new helixes of the same sized wire, but from a different factory, replaced the old ones. The surface of the helix removed from the dielectric tube was black, while that from the air tube was still bright. This was another proof that the alcohols had acted chemically on the wire. Each of the new helices was made from 36 feet of polished copper wire. The copper plug H of tube D (Fig. 1) was also insulated from the thermometer tube by a piece of vulcanite rod. With this change the platinum thermometer observations could be taken even if the tube D did contain an electrolyte.

With this new set of wires the resistance of W_1 was 0.24150 and of W_2 0.240439 in air.

The effect of sulphuric ether vapor on this new helix was also observed, and then the same absolute alcohol as before tested was again employed. Correcting for temperature, according to the platinum thermometer results, the effect of the alcohol was an increase of 0.008% over the first air value, but a decrease of 0.006% compared with the later air value. Chemical action and conductivity account for these.

Using Squibb's absolute alcohol, the specific resistance of which was 90,000 ohms, the result gave an increase of 0.002%. The chemical effect of this sample of alcohol was almost *nil.*

The results of these tests are summarized in the following table, Sanford's results for wire C_1 being also given. The helix first used is designated C_1 and the last C_2:

Dielectric	Sanford	Merrill		
		wire		Hours d'l't'c in tube
Kerosene	— 0.18%	C_1	+ 0.002%	53
Abs. Alcohol	+ 0.18%	C_1	— 0.001%, 1″ test } No temperature — 0.001%, 2″ test } Corrections made	17 17
		C_2	+ 0.008% or — 0.006%, 1″ sample + 0.002%, Squibbs	27 19
Abs. Alcohol Vapor	+ 0.05%	C_1	— 0.001%, 1″ test 0.000%, 2″ test	5.5 4
Sulphuric Ether Vapor	+ 0.25%	C_1	— 0.006%, 1″ test + 0.002%, 2″ test	19 40
Gasoline Vapor	+ 0.18%	C_2	+ 0.003%	23
		C_2	— 0.005%	45
Benzine (Petrolic Ether)	+ 0.06%	C_2	+ 0.005%	21
Benzine Vapor	———	C_2	— 0.002%	12
Chloroform Vapor	+ 0.17%	C_2	+ 0.002%	16

A "vapor" was merely air saturated with the true vapor, as was the case in Sanford's tests, except his gasoline vapor was a laboratory burning gas.

With the apparatus employed the experimental error was not greater than 0.01%; *and outside this limit the dielectric did not change the conductivity.* But I used a *polished* coiled wire, whereas Sanford used straight wires with surfaces more or less oxidized. Drs. Grimaldi and Platania also used a straight wire, but the condition of the surface was not stated.

But I am not strongly impressed with the accuracy of the quantitative results of Drs. Grimaldi and Platania, since they were deduced from small fractions of a mm. changes in galvanometer deflections. It is difficult to see how they could, by the ordinary telescope

and scale method, accurately estimate deflections to $\frac{1}{100}$ of a scale division, *i. e.*, to $\frac{1}{50}$ mm. Further, if the effect of petroleum (kerosene), as they found it, is deduced from the observations made while the wire was in petroleum and after the petroleum was run out, the result is only 0.010% instead of 0.015%. But their many careful observations seem to show that the kerosene did slightly decrease the resistance.

However, it is hard to see how the effect on a coiled wire should be different from that on a straight one. But since Ohm's law applies to a steady current in a helix, my results seem to show that, for this case at least, no modification in the law is needed.

The apparatus used was devised in this Laboratory, but the measurements were made at the University of Chicago during the summer of 1897.

I desire here to thank Professors Rowland and Ames for their kindly interest in my work, and Professors Michelson and Stratton for extending to me the privileges of the Ryerson Physical Laboratory.

PHYSICAL LABORATORY, JOHNS HOPKINS UNIVERSITY.

NEW BOOKS.

Harper's Scientific Memoirs. Edited by J. S. AMES, Ph.D. No. 1. The Free Expansion of Gases, 8vo, pp. v+106; No. 2. Prismatic and Diffraction Spectra, 8vo, pp. v+68. Harper & Brothers, New York, 1898.

These little volumes, the first of which contains a translation of Gay-Lussac's memoir by J. S. Ames, together with a reprint of the papers of Thomson and Joule on "Thermal Effects of Elastic Fluids," are the opening numbers of a series of reprints and translations by means of which the same service is to be rendered to the American scientific public that is being rendered in Germany by the Ostwald reprints of classical scientific memoirs. The selection of Gay-Lussac's paper was a happy one, since this important contribution is not readily accessible in the original. It is pleasure to have Gay-Lussac's work in the same volume with the later papers of Joule and Thomson.

The second volume of the series contains in addition to Fraunhofer's classical work the brief paper in which Wolliston, in the Philosophical Transactions for 1802, describes his method of examining refractive and dispersive powers by prismatic reflection. The introduction and many descriptions of experiments in the course of Fraunhofer's long paper, including, for example, ten pages on the description of experiments on different kinds of glass, are omitted. By thus cutting the bulk of the paper down and throwing out explanations and discussions here and there, the memoir is undoubtedly rendered much more readable than in the original form. Is there not, however, danger of creating a feeling of dissatisfaction on the part of the reader who finds he has not the whole of the author's treatise under his eye, and who perhaps, would prefer to omit for himself those parts in which he feels no interest? The only good reason that one can find for such excision is that of keeping the volume within a given number of pages.

It is to be hoped that these volumes will be followed by many others which will place, in convenient form, the most important of the classical treatises in physics at the disposal of American readers. While it is true that nearly all men of science in these days read French and German freely, it is likewise unfortunately true that many of them do not have access to the complete sets of European transactions. On this account a well selected series of translations and reprints will find a welcome place in the libraries of many physicists.

E. L. N.